Binary Numbers & Different Sizes of Infinity

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We've gotta have some ambitions!

Primary goal:

- prove there are at least two different sizes of infinity.

Here's the plan:

- First Half: binary numbers
- Second Half: sizes of sets

Controversy over different sizes of infinity

We will use Cantor's *diagonalization argument* (published in 1891). It will hopefully **blow your mind**, as it did mathematicians from the past:

> "I don't know what predominates in Cantor's Theory – philosophy or theology – but it has no proper place in mathematics." -Kronecker

> "Notion of a completed infinity does not belong in mathematics" -Gauss

Conclusion

We've proven that there is an infinite set that is BIGGER than the set $\{1, 2, 3, 4, \ldots\}$! So we've found at least two different sizes of infinity.

Fact: there are *uncountably* many different sizes of infinity. In other words, the set containing all sizes of infinity is also BIGGER than the set $\{1, 2, 3, 4, \ldots\}$.

The **Continuum Hypothesis** is a famous unknown conjecture in mathematics. It says the following:

> *There is no set whose cardinality is BIGGER than* $\{1, 2, 3, \ldots\}$ *but SMALLER than* $\{0, 1\}^\infty$*. In other words, there is nothing in between these two sets.*

SOME REALLY HARD EXERCISES...

1. Show that the set containing all real numbers is the same size as the set $\{0, 1\}^\infty$.

2. Look up how to construct the *power set* of any set. Adapt Cantor's diagonalization argument to prove that taking the power set of a set always produces a larger set. Show that the power set of $\{1, 2, 3, 4, \ldots\}$ is also the same size as $\{0, 1\}^\infty$.

3. Look up the **Generalized Continuum Hypothesis**.

4. Look up **Russell's Paradox**. It's just another version of Cantor's diagonalization argument, but it turned mathematics on its head.