Binary Numbers & Different Sizes of Infinity

SECTION 1

KEY: heads = 0 and tails = 1

(1) Drop two pennies so that they fall randomly and place them next to each other. Using the key above, this defines a binary number. Convert this binary number to a base 10 number. (In other words, write the binary number as a regular ol' number!). Repeat until you feel like you've got it.

example: $\text{tails} + \text{tails} = 11_{\text{bi}} = 1 \times (2^1) + 1 \times (2^0) = 2 + 1 = 3.$

(2) Now do the same exercise for three pennies. Can you do it for four pennies?

(3) What is the largest number you can make with two pennies? with three pennies? with four pennies?

(4) **Challenge:** How many different numbers can be made with two pennies? with three pennies? with four pennies?

(5) **Extra Challenge:** Convert the following numbers to binary: 4, 10, 18, 27, 52. You might find this information helpful:

$$2^0 = 1, \quad 2^1 = 2, \quad 2^2 = 4, \quad 2^3 = 8, \quad 2^4 = 16, \quad 2^5 = 32, \quad 2^6 = 64$$

SECTION 2

(1) Are the following sets the same size? Prove your answer.

$$\{1, 2, 3, 4, 5, 6, 7, \ldots\} \quad \text{and} \quad \{9, 10, 11, 12, 13, 14, 15, \ldots\}$$

(2) Can you find an element of $\{0, 1\}^\infty$ that isn't on the list? Why does this prove that the two sets are different sizes? Why is it called a "diagonalization argument"?

(3) **Challenge** Suppose you are given a list of n binary numbers that can each be made with n pennies. Can you write down a method for constructing another binary number that is not on the list but can also be made with n pennies? (You might need to use the back of the page for scratch paper!)