Pigeonhole Problems
(For all of the problems, think about what are the pigeons and what are the holes.)

1. If you have only two colors of socks – white and black – and you grab three socks, you are guaranteed to have a matching pair.

2. Suppose no Texan has more than 200,000 hairs on his or her head. There are at least 120 Texans with exactly the same number of head hairs.

3. Suppose $S$ is a set of 8 integers. There exist two distinct elements of $S$ whose difference is a multiple of 7.

4. Among any group of six acquaintances there is either a subgroup of three mutual friends or three mutual enemies.

5. Given twelve coins – exactly eleven of which have equal weight - determine which coin is different and whether it is heavy or light in a minimal number of weighings using a three position balance.

6. Given seven coins such that exactly five of the coins have equal weight and each of the other two coins is different – possibly heavy or lighter. To determine which coins are different and whether each different coin is heavy or light requires at least five weighings using a three position balance.

7. Given five points inside an equilateral triangle of side length 2, at least two of the points are within 1 unit distance from each other.

8. In any sequence of $n^2 + 1$ distinct integers, there is a subsequence of length $n+1$ that is either strictly increasing or strictly decreasing.