

COMPLEX ANALYSIS – HOMEWORK ASSIGNMENT 8

Due Friday, March 29, 2013, at the beginning of class.

Please write clearly, and staple your work !

1. PROBLEM

Consider the Weierstrass factor of order m , $E_m(z) = (1 - z) \exp(\sum_{\ell=1}^m \frac{z^\ell}{\ell})$.

(i) Prove that

$$|E_m(z) - 1| \leq |z|^{m+1} \quad \text{for all } z \in \mathbb{D}.$$

Hint: Consider the logarithmic derivative $\frac{d}{dz} \text{Log} E_m(z)$.

(ii) Prove that

$$\ln(|E_m(z)|) \leq (2m + 1)|z|^{m+1} \quad \text{for all } z \in \mathbb{C}.$$

2. PROBLEM

Assume that $\{z_j\}_{j \in \mathbb{N}} \subset \mathbb{C}^*$ is a sequence with no accumulation points. Consider the infinite product

$$f(z) = \prod_{j=1}^{\infty} E_{m_j} \left(\frac{z}{z_j} \right).$$

(i) Prove that the product converges for all z , and defines an entire function for $m_j = j$.

(ii) Assume that $\sum_j \frac{1}{|z_j|^{p+1}} < \infty$. Prove that the product converges for all z , and defines an entire function for $m_j = p$ (independent of j).

3. PROBLEM

Show that for any positive real number r ,

$$f(z) = \sum_{n=0}^{\infty} \frac{z^n}{(n!)^{1/r}}$$

defines an entire function f of order r .

4. PROBLEM

Find all entire functions f that satisfy $f(\sqrt{n}) = n^2$ for every positive integer n , and $|f(z)| \leq e^{3|z|}$ for every complex number z .