Open problems

The Art of a Commuting Pair

Introduction

Outline of this talk

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Linear operators

Aluthge and Mean transforms of bounded

1
For \( \text{Te}(B) \), the polar decomposition of \( B(\alpha) \): Algebra of bounded operators on the complex Hilbert space.

Introduction
Complex matrix of a square

The polar decomposition of a square.

The complex value of $z$ of $z = r \cdot e^{i\theta}$ is $|z| = r \cdot e^{i\theta}$.

Examples.

- Zero complex number:
  - The polar form (decomposition) of $a$.

- Unitary matrix $U$:
  - Hermitian matrix $H$.

- $A = I + T$.
\[ A \notin \text{ invertible} \implies \log (\mathbf{T} \mathbf{T}^*) \geq \log (\mathbf{T} \mathbf{T}^*) \]

\[ \text{for } a > p \geq 1 \]

Study p-hermoparmonic \( \neq \log \)-hyper. oper.

Aluthage in [1990], \text{IEEE}, 1990, in order to

\[ \text{This transform was first studied by} \]

\[ \text{The Aluthage transform of } \mathbf{E} \text{ is} \]

\[ \text{Hermitian matrix} \]

\[ \begin{pmatrix}
\frac{1}{2} & \frac{1}{2} \\
\frac{1}{2} & -\frac{1}{2}
\end{pmatrix}
\]

\[ =
\begin{pmatrix}
1 & 0 \\
0 & 0
\end{pmatrix}
\]
Does I have a nontrivial (\neq 0, x) \& \text{Te} \text{b(x)}.

Let \text{x} be a Banach space of dim x \geq 2

\text{Invention subspace problem: } \text{Invention subspace problem:}

\text{Problem connection with the invention subspace}

\text{One reason is its in recent years. The art has received much attention.}
Basic results of the AT

Jung, Ko and Pearcy proved in [IIEOT, 2000] that for Te, B, T has a nontrivial invariant subspace $\iff T$ does not preserve the spectrum of $T$ in $[IIEOT, 12]$ equals $T_{EOT}$ that AT need not preserve the $k$-hyponormality for $k \geq 2$.
A normal operator is closer to being of an operator is also known that the iterated AT

It is well known that T has a nontrivial invariant subspace if T is normal.

\( \begin{align*}
\frac{1}{2} (x) & = (\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}) \\
\frac{1}{2} (2) & = (\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}) \\
\frac{1}{2} (1) & = (\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}) \\
\frac{1}{2} (0) & = (\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}) \\
\end{align*} \)
For a weighted shift

\[ \omega_d \equiv \text{shift} \left( d_0, d_1, d_2, \ldots \right) : \ell^2(\mathbb{Z}^+) \rightarrow \ell^2(\mathbb{Z}^+), \]

\[ \omega_d = U + D_d = \begin{pmatrix} 0 & & & \\ \vdots & \ddots & \ddots & \\ & & 0 & \vdots \\ 1 & 0 & & 0 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ 0 \end{pmatrix}, \]

where \( x_i > 0 \).

\( \tilde{\omega}_d = D_d^{1/2} U + D_d^{1/2} = \text{shift} \left( \sqrt{d_0 d_1}, \sqrt{d_1 d_2}, \sqrt{d_2 d_3}, \ldots \right) \)

\( \therefore \omega_d (e_n) = D_d^{1/2} U (e_n) = D_d^{1/2} \left( \sqrt{d_n} e_n \right) = \sqrt{d_n} \cdot D_d^{1/2} (e_{n+1}) = \sqrt{d_n} \cdot D_d^{1/2} (e_{n+1}) = \sqrt{d_n d_{n+1}} \cdot e_{n+1} \)
Shift \( \frac{2}{4} \), it converges to \( \frac{2}{4} \) at \( \lambda \in B(\alpha) \),

\[ \frac{1}{3} \leq \text{Motivation of the Mean Transformation} \]

Because it involves the term \( \frac{1}{3} \),

\[ \frac{1}{3} \leq \text{Involves the term} \quad \frac{1}{3} \leq \text{Involves} \quad \frac{1}{3} \leq \text{Involves} \quad \frac{1}{3} \leq \text{Involves} \]

It is so hard to find the actual use,

In the view of the practical use,
Polar decomposition of operators.

- If we know the

\[
\frac{z}{2}, \frac{z}{2+\alpha}, \ldots
\]

\[
\Rightarrow \text{shift} (\alpha_0 + x_1, \alpha_0)
\]

- For \( A_1 \equiv \text{shift} (\alpha_0, \alpha_0, \ldots) \)

Dual diagonal transformation.

\[
1 \leftarrow 1 \equiv -\frac{1}{2} (211 + 111) \quad \text{where}
\]

\[
1 = -\frac{1}{2} (211 + 111)
\]

by the mean of \( t \), then we define the mean of \( t \) as the polar decomposition.

\[
\Rightarrow 1 \leftarrow 1 = 111
\]
The $\mathcal{B}(\mathcal{A})$ is subnormal

For $T \in \text{normed}$

$T \in \mathcal{B}(\mathcal{A}) \implies$ jointly hyponormal

The $\mathcal{B}(\mathcal{A})$ is $k$-hyponormal if $(I, T, T^2, \ldots, T^k)$

The $\mathcal{B}(\mathcal{A})$ is hyponormal if $T^* T \geq T T^*$

The $\mathcal{B}(\mathcal{A})$ is hyponormal if $T^* T \geq T T^*$

The $\mathcal{B}(\mathcal{A})$ is subnormal if $T^* T \geq T T^*$

The $\mathcal{B}(\mathcal{A})$ is normed if $T = N|T|
eq 0$

Let say that

$\forall$
If \( \lim_{t \to 0} f(t) = 0 \) for each \( x \), \( \lim_{t \to 0} (x)_{f(t)} = 0 \) \( \lim_{t \to 0} \|x\|_{f(t)} \leq \|x\|_0 \) \( \forall (x) \in B(c(t)) \). 

That is, \( \lim_{t \to 0} \) - continuous at \( B(c(t)) \).

The mean transformation map \( T \) is is a positive operator.

Then \( f(1) = 1 \) \( f(t) = \phi(t) \) \( \phi \in B(c(t)) \) when \( \phi \in B(c(t)) \).

Ex: \( T : \{0, p\} \in B(c(t)) \) where \( p \in B(c(t)) \).

The spectrum of \( T \) is not equal to that of \( T \).

In \([0, 13], \) we show the following:

① Basic results of the MT.
If \( P \Rightarrow Q \) is true, then \( P \) can always be considered true for any \( Q \) that is true.

\[ \text{If } \neg P \Rightarrow \neg Q \text{ is not true, then } \neg P \text{ is not true.} \]

Example: \( P = \text{it is sunny} \), \( Q = \text{it is not cloudy} \).

Then we have:

\[ \neg P = \neg \text{it is sunny} \text{ but } \neg Q = \text{it is not cloudy} \text{ is true.} \]

But the converse of it is not true.

The MT \( \neg P \not\rightarrow \neg Q \) is also not true.
And plausible definition

Thus, we don't consider this one.

\[
\frac{\text{t} - \sqrt{\text{t}^2 + 1}}{\text{t}} \land \text{t}_2 = \text{t}_1 \sqrt{\text{t}_1^2 + 1}
\]

The following:

Isometry \( L \) which satisfies simultaneously

But we can't find a common point

\[
\text{t} : = (\text{t}_1, \text{t}_2) : \left( \frac{\text{t}_1^2 + 1}{\text{t}^2 + 1}, \frac{\text{t}_1^2 + 1}{\text{t}^2 + 1}, \frac{\text{t}_1^2 + 1}{\text{t}^2 + 1} \right)
\]

\( \text{t} \) at a common point of \((\text{t}_1, \text{t}_2)\)
so we don't consider this one, not hyper.

\[ F \in \mathcal{L}(V, V) \] \( \iff \) \( l \in \text{hyper} \), but \( l \neq \text{hyper} \)

\[ \text{ker}(l) \subseteq \text{ker}(l') \]

\[ \text{ker}(l) \bigcap \text{ker}(l') = \{0\} \]

that is, \( \text{ker}(l) \bigcap \text{ker}(l') = \{0\} \). (point) partition isometry.

\( (l', l) = \) \( (l', l,) = (l, l') \). (point) partition is not the AR of \( l \), but

\[ l' = l \bigcap l' \]

Then a polar decomposition of the pair \((l', l)\).

\[ \text{where } P = l - l' \]

where \( P = l - l' \).
Problem 2. Does a Taylor spectrum of \( U \) equal that of \( U + \psi \) ?

\( \psi \) is a \( k \)-hyperpoles, does it follow that \( AT \) is \( \psi \)?

Problem 1. For \( k \geq 1 \), if \( \psi \) is \( \psi \), then \( AT \) is \( \psi \).

We study the following two problems.

Based on the definition of \( AT \),

\[ \psi \equiv (\psi, \psi) : = (1/n, 1/n), 1/n \in \mathbb{R} \]
C. \( \text{the class of } k \)-hyper. pairs in \( C_0(k,2) \)

- \( \text{operators on a Hilbert space } \mathcal{H}^2 \)
- \( \text{the class of commuting pairs of } \mathcal{H}^2 \)

For \( k \in \mathbb{R}^+ \) and \( k \neq k' \):

\[ \mathcal{E} = (1, 0), \quad \mathcal{E} = (0, 2) \]

- \( \mathcal{T}_L(\mathcal{E}, k) = \{ g \in \mathcal{K}, k \} \in \mathcal{K}(k, k') \)
- \( \mathcal{T}_R(\mathcal{E}, k) = \{ \alpha \in \mathcal{K}, k \} \in \mathcal{K}(k, k') \)

such that:

\[ \mathbb{R}^2 \rightarrow \{ (x, y) \in \mathbb{R} \} \]

\( \mathcal{X} \)-variance weighted shift

\( \mathcal{Y} \)
- $C_\infty$: the class of subnormal pairs in $C_0$.
- We have that $C_\infty \leq \ldots \leq C_k \leq \ldots \leq C_1 \leq C_0$.
- Weight diagram of $\omega(\alpha, \beta) \equiv (T_1, T_2)$
\[ \begin{pmatrix} \ast & \ast & \cdots & \ast \\ \vdots & \vdots & & \vdots \\ \ast & \ast & \cdots & \ast \\ \ast & \ast & \cdots & \ast \end{pmatrix} \begin{pmatrix} \ast \\ \vdots \\ \ast \\ \ast \end{pmatrix} \iff \text{(finiteness)} \land \psi \land \left( \exists \chi \in \text{ (finiteness)} \land \chi \right) \land \psi \land \left( \exists \chi \in \text{ (finiteness)} \land \chi \right) \land \psi \land \left( \exists \chi \in \text{ (finiteness)} \land \chi \right) \land \psi
\[\text{If } C \cap A \cap R \neq \emptyset, \text{ we have that for } k, k_2 > 0\]

\[\Rightarrow 0 \leq (k, k_2)\]
The given table is nonexistent.

By Figure 2
\\( P_{4}^{x} = \frac{t}{3} \)

If \( \text{c}(x,p') \) is given,

Thm 3:

If \( \text{c}(x,p') \neq C \)

E.

Thm 2:
A cochain complex (called the Koszul complex), denoted by

\[ K(\Delta, x) : 0 \to D_x \to \cdots \to D_x \to D_x \to 0 \]

where the restriction of \( D_x \) to \( x \) is

\[ \text{antichain operator} \]

\[ D_x \to x \to \cdots \to 0 \to D_z \to \cdots \to D_x \]
• If \( \text{Ran } D^\Delta \neq \ker D^\Delta \), then the Koszul complex is said to be *exact*.

• \( \Sigma_T(\Delta) := \{ (\lambda_1, \lambda_2) \in \mathbb{C}^2 \mid \text{K}( (T_1 - \lambda_1, T_2 - \lambda_2), \mathbb{C}) \text{ is not exact} \} \)

5 Open problems

• This work is only a start on the theory of the MT (resp. AT of a commuting pair) of bounded operators

0 If \( T \) is log-hyponormal, is \( T \) log-hypo.? the MT
I has a nontrivial invariant subspace

\[ f \left( \| x \| \right) \xrightarrow{\text{if}} \lim_{n \to \infty} \frac{1}{n} \sum_{k=1}^{n} f^k(\| x \|) = 0 \]

That is, \( \| x \| \to 0 \) - continuous on \( B(x_0) \)

Is the mean transformation map \( T \)

Also p-phylo. i.e., for any \( p > 1 \), if \( T \) is p-

"penomological, \( \alpha \)
Thank you.

If a common nontrivial subspace $(\Pi, T_{2})$ has a common nontrivial subspace $(\Pi, T_{1})$ has a common nontrivial subspace.

That is (or is it subnormal?) does it follow that $C(a, b)$ is subnormal, does it follow.

\[
\frac{1}{2} (\frac{1}{2}, \Pi_{2}) \sim \frac{1}{2} (\frac{1}{2}, \Pi_{1}) = 0 = 0 \sim b (\frac{1}{2}, \Pi_{b}) \sim \frac{1}{2} (\frac{1}{2}, \Pi_{a,b})
\]