Abstracts presented to the 33 Texas PDE seminar

Austin TX April 10, 11 2010

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Stochastic Modeling of Multistage Carcinogenesis Mutations

Oncogene accelerates cell division and tumor suppressor gene slow it down. Both play important role in developing cancer tumor. Initially we accept Hardy-Weinberg assumption which implies that each genotype for tumor suppressor gene T and oncogene O that is TT TOC OO is equally fit. That means the expected number of genes an individual contributes to the gene pool of the next generation is independent of genotype. A stochastic model for the interactions between the tumor suppressor gene T and the oncogene O is developed. The gene pool can be described from one generation to the next by the sequence p(n) where p(n) denotes the frequency of the tumor suppressor gene pool of type T immediately before the n reproduction. We design the stochastic interaction function of multistage mutation based on the formula in our model. In this formulation the tumor suppressor gene has a selective advantage. We assume genotypes have different selective factors and design our interaction between stages using a population genetic model.

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Orthogonal Bases and Reproducing Kernels for Hilbert Spaces of Harmonic Functions

The space of finite energy H^1-harmonic functions on a region Ω has a basis of Steklov eigenfunctions that are L^2-orthogonal on the boundary and have orthogonal gradients on the region. This leads to a constructive characterization of the boundary trace spaces $H^{1/2}(\partial\Omega)$ for the region with explicit formulae for inner products and some standard operators. In particular there are general formulae for the Poisson kernel and for the solution operators for Robin and Neumann boundary value problems on the region Ω. This analysis is extended to a 1-parameter family of trace spaces $H^s(\partial\Omega)$ and their harmonic extensions $\mathcal{H}^{s+1/2}(\Omega)$. We shall show that, provided the boundary data is L^2, the corresponding spaces of harmonic functions on Ω are reproducing kernel Hilbert spaces with respect to a natural inner product. The reproducing kernels are found explicitly via the Steklov eigenfunctions of the domain. This extends earlier results of Peetre et al and J.L. Lions.

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Nonlinear Magneto-Convection in a Passive Mushy Layer

Here we consider solidification of a binary alloy cooled from below. It has been well established by experimentalists that a horizontal mushy layer is formed during the solidification process. This study investigates nonlinear behavior of the convective flow in the mushy layer in the presence of a magnetic field. We derive linear, adjoint and first-orders systems. We also derive the evolution equation of Landau type for the amplitude. Using numerical methods, the Landau constants are computed. The numerical results for marginal stability curve, linear and first-order solutions related to the vertical velocity component and the solid volume fraction and amplitudes are presented. Joint work with Daniel N. Riahi.

Guy Bernard
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Global Existence to the Navier-Stokes Equations Through a Self-Similar-Like Upper Solution.

A global existence result is presented for the Navier-Stokes Equations filling out all of three dimensional space. The initial velocity is required to have a bell-like form. The method of proof is based on the symmetry transformations of the Navier-Stokes equations and a specific upper solution to the heat equation in the time interval $[0, 1]$. This upper solution has a self-similar-like form and models the diffusion process of the heat operator. This demonstration technique avoids the use of weak solutions as it establishes the existence of regular solutions directly. It consists of an iterative process where existence is demonstrated in a sequence of time intervals. In each extended interval, the Navier-Stokes problem is transformed into the local one in the time interval $[0, 1]$ by a symmetry transformation which exploits the diffusive nature of the upper solution.

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Computing the Minimal Average Energy: Numerical and Perturbative Results

We use the steepest descent technique called "Sobolev gradients" to numerically compute plane-like minimizers, $u_\omega(x) = \omega \cdot x + v(x)$ of the variational problem $S(u) = \int \frac{1}{2}|\nabla u| + V(x,u)\,dx$, where $\omega \in \mathbb{R}^d$, v periodic. The minimal average energy $A(\omega) = \lim_{r\to\infty} \frac{1}{|B_r|}\int_{B_r} \frac{1}{2}|\nabla u_\omega| + V(x,u_\omega)\,dx$ is a well-defined function of the average slope ω. We investigate the jumps in $\nabla A(\omega)$ numerically, and compare with results using perturbative techniques. A has the physical interpretation as the surface tension of a crystal face, and the differential properties of A determine the shape of the crystal.

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High Field limits for magnetized plasmas.

Many research projects in plasma physics concern the energy production through thermo-nuclear fusion. The controlled fusion requires the confinement of the plasma into a bounded domain and for this we appeal to the magnetic confinement. Several models exist for describing the evolution of strongly magnetized plasmas : the guiding-center approximation, the finite Larmor radius regime, etc. The subject matter of this talk is to present asymptotic models in the general three dimensional setting under the action of large stationary inhomogeneous magnetic fields. The mathematical analysis relies on average techniques, related to ergodic theory and homogenization procedures. We propose a general method providing rigorous derivations for such models. One of the key points is to observe that the orthogonal projection on the kernel of a linear transport operator is given by the average along its characteristic flow. We use here the mean ergodic theorem of von Neumann. The computations simplify a lot when appropriate coordinates are choosen. In particular, to any prime integral of the dominant characteristic flow, we associate a derivation commuting with the average operator. At the leading order the particles are advected along the magnetic lines. The plasma remains confined around the magnetic lines. But perpendicular drifts occur at the next order (electric cross field drift, magnetic gradient drift, magnetic curvature drift), which may destroy the confinement. Therefore it is very important to compute them, in order to estimate the confinement time. Using similar techniques we can take into account the first order corrections and finally we propose a second order accurate model for the gyro-kinetic Vlasov equation.

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Porous media equation with non local pressure

we will discuss the properties of solutions to a porous media type equation where the pressure is not a pointwise function of the density but is determined through the full density distribution through a potential.

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Superposition Principle in Finding Topological Derivatives

Superposition principle is used in finding topological derivatives for elliptic partial differential equations. The superposition principle is applied to decompose both the solutions and boundary conditions (BCs). Two type of cost functions are investigated here. By using superposition principle we can handle Dirichlet, Neumann, and Robin BCs, and we also gain more insight of finding topological derivatives. Joint work with Glaucio H. Paulino

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Nonlinear Water Wave Equations on a Running Stream near Resonant Conditions

This paper deals with the surface water waves on a running stream of velocity U and depth h. There exist two resonant conditions: (i) $U = U_c = \sqrt{gh}$ associated with the steady pressure distribution and (ii) $U = U_c = \frac{g}{4\omega}$ associated with the pressure distribution of frequency ω, applied to the free surface for the generation and propagation of water waves where g is the acceleration due to gravity. The linearized water wave equations admit unbounded solutions near resonant conditions. This paper deals with the nonlinear water wave equations near resonant conditions.

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Invariant manifolds for bi-semigroups

We show the existence of local Lipschitzian stable and unstable manifolds for the ill posed problem of perturbations of hyperbolic bisemigroups without assuming backward nor forward uniqueness of solutions and without assuming global smallness conditions on the nonlinearities. We introduce what we call /dichotomous flows/ which recover the symmetry between the past and the future. We use the /Conley-McGehee/ approach which fits very well with the /dichotomous flows/ formulation.

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Integrable Discretizations and Self-adaptive Moving Mesh Method

The study of integrable discrete systems opens a most promising way towards the general theory of difference equations or discrete systems. In the present talk, we will report some progress in our series work of obtaining integrable analogues for a class nonlinear PDEs possessing cuspon or loop soliton solutions. These equations, which include the Camassa-Holm, the short pulse and the Hunter-Saxton equations, are important equations in physics.

Firstly, we will clarify the connection between these equations and some well-known integrable systems such as the two-dimensional Toda-lattices (2DTL) through hodograph transformation. Then, integrable discretizations are constructed by the Bäcklund transformation of 2DTL and defining appropriate discrete hodograph transformations. Meanwhile, the multi-cuspon or multi-loop solutions are given in the form of Casorati determinant. Finally, we will show the resulting integrable discrete systems can be served as an innovative numerical scheme, i.e., a self-adaptive moving mesh method for the numerical simulations of these PDEs.

This is a joint work with my colleague Dr. Kenichi Maruno and Dr. Yasuhiro Ohta at Kobe University of Japan.

Juan Galvis
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Numerical homogenization for high-contrast elliptic problems using local spectral basis functions

We study numerical homogenization with the multiscale finite element methods (MsFEMs) designed for high-contrast problems. The new nultiscale basis functions are constructed using eigenvectors of a carefully selected local spectral problem. The local eigenvalue problem automatically detects high-conducting components of the solutions via small, asymptotically vanishing, eigenvalues. Multiscale basis functions span (locally) the small eigenvalue eigenfunctions. Our numerical results show that the new MsFEMs constructed via local spectral problems are more accurate compared to multiscale methods that employ traditional multiscale spaces with one basis function per coarse node. The numerical results are presented

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Classical solutions for the Boltzmann transport equations for soft potentials with initial data near local Maxwellians.

We present first show convolution inequalities for the Boltzmann collision operator. More specifically we show that a symmetric convolution structure on the gain part of the collision operator yields Young's inequality for hard potentials and a corresponding Hardy Littlewood Sobolev inequality for soft potentials. The constant are computed exactly depending on the integration of the scattering cross section.

Second, we revisit the existence and uniqueness theory of the Boltzmann equation for soft potentials with integrable scattering cross section (Grad cut-off) and initial states near local Maxwellians distributions. Using the derived convolution estimates, we show that the solution is classical if initially so and L^p stable for $1 <= p <= n/n - b$ where b is soft potential exponent.

This is work in collaboration with Ricardo Alonso and partly with Emanuel Carneiro

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Can the two-layer QG model explain the Nastrom-Gage energy spectrum of the atmosphere?

An analysis of wind and temperature measurements taken during the Global Atmospheric Sampling Program by Nastrom and Gage showed that there is a robust k^{-3} energy spectrum extending from approximately 3,000 km to 1,000 km in wavelength and a robust $k^{-5/3}$ energy spectrum extending from 600 km down to a few kilometers. Tung and Orlando have demonstrated numerically that the two-layer quasi-geostrophic model, forced at large scales by baroclinic instability, can reproduce this energy spectrum. However, their result is considered, by some, to be controversial. More detailed models have been shown to reproduce the Nastrom-Gage energy spectrum as well. However, the question remains: why do any of these models work, and what is the simplest model that can account for the Nastrom-Gage spectrum? In this talk I will present the recent progress that has been made towards addressing these questions.

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L-infinity estimates for gradients of solutions to some nonlinear problems

In this talk a general framework allowing for L-infinity estimates for gradients of solutions to a class of nonlinear PDE problems, whose prototype is the p-Laplacian, will be presented. The main goal of the developed approach is to capture and characterize the blow up of the electric field in a two-phase high contrast material consisting of a matrix with two injected particles close to touching. The described approach is of the asymptotic nature. More specifically, the quantities of interest are obtained asymptotically as the small parameter of the problem, which is the order of the distance between neighboring particles, is close to zero. Such an approach provides a basis for developing new techniques to attack the nonlinear case, in which key features of the class of problems of interest are taken into account, namely: the small distance between particles, and high contrast in mechanical properties of composite constituents. Those specific features lead to localization of the so-called high concentration zones where gradients of solutions to the corresponding problems exhibit singular behavior.

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Instantaneous regularization phenomena for the Stefan problem with Gibbs-Thomson law

The two-phase Stefan problem is expected to exhibit waiting time phenomena, in other words, an initial Lipschitz free boundary might remain just Lipschitz for some positive time. On the other hand, adding a Gibbs-Thomson correction term to the model is believed to stabilize the interface and make it more regular, I will present a partial result in this direction, namely, I will show that free boundaries which are Lipschitz in space and time are actually $C^{2,\alpha}$ in space and become so instantaneously. This will follow first from observing that the De Giorgi-Moser-Nash theory for parabolic equations works even when one has a singular right hand side and secondly from the regularity theory of almost-minimal boundaries of Almgren, De Giorgi and Tamanini.

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PDE model of the optimal forest management with carbon sequestration and environmental impact

A model for the optimal forest management is presented as a boundary value problem for nonlinear partial differential equations with integral terms. The model takes into account the size structure of trees, intra-species competition, and density effects as well as environmental impacts on biological parameters of trees and economic values. The model includes soil carbon concentration as a separate stock variable and considers changes of parameters as consequence of climate change. Delay differential equations are employed to describe delayed processes of carbon sequestration. The objective function takes into account the revenue from timber production, operational expenses, and the profit from carbon sequestration. The model is calibrated on real data about government forest management in Spain. The dynamics of climate change is taken from known global scenarios and incorporated in the model. The dual system is derived and a maximum principle is obtained. Provided qualitative analysis of the formulated optimization problem leads to better understanding of how environmental changes impact biological processes of forest, carbon sequestration, and mitigation costs; how carbon price and the optimal logging time change within climate changes; optimal management of carbon sequestration and timber production adapted to climate changes. Joint work with Yuri Yatsenkob, Renan-Ulrich Goetzc, Angels Xabadia

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Rigorous computation of symmetric connecting orbits for systems of second order ODE's.

I will discuss a scheme for computer assisted proof of the existance of certain symmetric heteroclinic and homoclinic orbits in systems of coupled second order ordinary differential equations. The idea is to formulate the connecting orbit as the solution of a free boundary problem, which we solve using a Newton Scheme and finite element methods. Boundary conditions at infinity are avoided by requiring that the solution is consistant with a high order parameterizations of the stable and unstable manifolds of the equilibria. The numerical solution is rigorously validated via an 'a posteriori' fixed point argument which depends on analytic control of the truncation error.

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A free boundary problem for the isentropic gas dynamics equations

We consider a two-dimensional Riemann problem for the isentropic gas dynamics equations leading to strong regular reflection. We rewrite the problem using the self-similar coordinates and we obtain a mixed type system and a free boundary problem for the subsonic state and the reflected shock. Using the theory of second order elliptic equations and various fixed point arguments, we prove local existence of a solution.

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Quasi-periodic and almost periodic solutions in coupled map lattices

We show the persistence of some quasi-periodic oscillations in some discrete systems (modelling e.g. linear molecules). The method is a a-posteriori theorem of persistence. This allows us to show that in some models there are solutions with infinitely many frequencies. We note that these solutions cannot exist in the continuum approximation. Joint work with E. Fontich, P. Martın, Y. Sire.

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Hydromagnetic Waves in a Thin Rotating Spherical Shell

We consider an electrically conducting fluid confined to a thin rotating spherical shell in which the Elsasser and magnetic Reynolds numbers are assumed to be large which the Rossby number is assumed to vanish in an appropriate limit. This may be taken as a simple model for a possible stable layer at the top of the Earth's outer core. It may also be a model for the thin shells which are thought to be a source of the magnetic field of some planets such as Mercury or Uranus. Linear hydromagnetic waves are studied using a multiple scale asymptotic scheme in which boundary layers and the associated boundary conditions determine the structure of the waves. The results are used to study the stability of an ambient field as well as to compare the behavior of the waves to the case in which boundary conditions have been ignored

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Geometric formulation of motion of discrete curves and integrable self-adaptive mesh schemes for some PDEs

Continuous integrable systems such as the KdV equation in water waves have been known for many years. On the contrary, discrete integrable systems have only come to the fore in the last 2 decades. I will give a short review of this new field. Then I will talk about the geometric formulation of the motion of discrete curves which leads to "discrete" differential geometry. Using this geometric formulation, we can naturally obtain integrable self-adaptive mesh schemes of PDEs belonging to the WKI form (e.g. short pulse equation, Camassa-Holm equation, Harry Dym equation, etc.) which were recently proposed by us.

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Global vs. Local Existence for Time Dependent Equations

The problem of whether an autonomous nonlinear system, on a space X, has local or global existence in time is reduced to a linear eigenvalue problem on the space $CB(X)$ of bounded real-valued continuous functions on X. The start of a corresponding numerical attack on this problem is described.

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On the Regularity of Integral Variational Obstacle Problems

We will present some regularity results of integral variational obstacle problems. Firstly, we will talk about the origin of integral variational problems, and then some recent studies about their regularity theory. Lastly, we will present our studies about these problems with an obstacle.

Misha Perepelitsa
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The vanishing viscosity limit for the Navier-Stokes equations

In this talk we will discuss the vanishing viscosity limit of the Navier-Stokes equations to the isentropic Euler equations for one-dimensional compressible fluid flow. We will follow the approach of R.DiPerna (1983) and reduce the problem to the study of a measure-valued solution of the Euler equations, obtained as a limit of a sequence of the vanishing viscosity solutions. For a fixed pair (x,t), the (Young) measure representing the solution encodes the oscillations of the vanishing viscosity solutions near (x,t). The Tartar-Murat commutator relation with respect to two pairs of weak entropy-entropy flux kernels is used to show that the solution takes only Dirac mass values and thus it is a classical weak solution of the Euler equations. This is a joint work with Gui Qiang Chen (Oxford University and Northwestern University).

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Mathematical models for Inverse problem in small volume

Inverse problems in medical imaging are in their most general form ill-posed problems. They literally have no solution. If, however, in advance we have additional structural information or supply missing information, then we may be able to determine specific features about what we wish to image with a satisfactory resolution and accuracy. The diffusion approximation is commonly used for biomedical optical imaging in turbid large media where absorption is low compared to scattering system. Unfortunately, this approximation has significant limitations to accurately predict photon transport in turbid small media and also in a media where absorption is high compared to scattering systems. A radiative transport equation (RTE) is best suited for photon propagation in human tissues. However, such models are quite expensive computationally. To alleviate the problems of the high computational cost of RTE and inadequacies of the diffusion equation in a small volume, we use telegrapher equation (TE) in the frequency domain for fluorescence-enhanced optical tomography problems (inverse problem). The telegrapher equation can accurately and efficiently predict ballistic as well as diffusion- limited transport regimes which could simultaneously exist in small animals. The accuracy of telegrapher-based model is tested by comparing with the diffusion-based model using stimulated data in a small volume. This work demonstrates the use of the telegrapher-based model in small animal optical tomography problems.

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Special solutions of higher dimensional Frenkel-Kontorova model—Aubry-Mather theory in discrete elliptic PDEs

We talk about the standard Frenkel-Kontorova model, its generalization and the related Aubry-Mather theory. Under the standard assumptions, we prove the existence of minimal Birkhoff configurations with given frequency vector for the multidimensional discrete periodic variational problems of the generalized FK model. In addition, we prove the strong comparison principle of the hull functions.

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Spectral gaps of certain periodic Maxwell operators.

The Bethe-Sommerfeld conjecture states that the spectrum of the stationary Schroedinger operator with a periodic potential in dimensions higher than 1 has only finitely many gaps. After work done by many authors, it has been proven by now in full generality. We consider a similar conjecture for a periodic Maxwell operator. This operator is interesting due to its importance to the photonic crystals theory. In this context, existence of a spectral gap implies the photonic crystal prevents light from propagating at certain frequencies. The latter property finds a number of applications in optics and electronics.

It turns out that the case of Maxwell operator is more complicated than a Schroedinger operator. It is harder to open a spectral gap for such an operator and at the same time it is harder to show the absence of gaps.

We establish finiteness of the number of spectral gaps in the case of a $2D$ photonic crystal, i.e. for the medium periodic in two variables and homogeneous in the third one, provided that the dielectric function is separable. This is the first result of that kind. We also show that, as one should expect, when the medium is near to being homogeneous, there are no spectral gaps at all.

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On Solving Nonlinear Differential Eigen-Solution Problems

Many problems in the study of solution pattern, (in)stability analysis and other properties lead to solve a nonlinear eigen solution problem. Such problems are very difficult to solve due to their nonlinearity, multiplicity and non variational nature. Starting with a Schrodinger type nonlinear eigen solution problem, the speaker will address several types of nonlinear eigen solution problems, discuss their significant difference from linear eigen solution problems, propose their solution concepts and methods and present some basic results. Numerical examples will be shown to illustrate the methods. This is a joint work with Changchun Wang. This research is supported in part by NSF-DMS-0713872/0820327.
