M 375 – Homework 10

4.6.1

Parity check matrix *H*:

$$H = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

The columns of the parity check matrix are the binary representation of the number 1…15.

(a) syndrome = 0001 ⇒ first digit of codeword is wrong:
correct codeword (000 000 000 000 000).

(b) syndrome = 0000 ⇒ no error: (111 111 111 111 111).

4.7.1

There are 8 possible binary codes of length 3.

Those are $\{(000),(001),(010),(011),(100),(101),(110),(111)\}$. But only 4 of them are linearly independent: a) (000): 0
b) (001): 1
c) (011): $x+1$
d) (111): x^2+x+1

All other codes can be produced by cyclic permutation of a) – d):

(010) and (100): permute b) once and twice, respectively
(101) and (110): permute c) once and twice, respectively.

4.7.2

Known from the lecture: $g(x)h(x) = x^7 - 1$ and $g(x) = x^3 + x + 1$.

$(x^3+x+1)(x^4+x^2+x+1) = x^7+x^5+x^4+x^3+x^5+x^3+x^2+x+x^4+x^2+x+1$.

In $F_2^7[x]$ this polynomial is reduced to: $x^7 - 1$.