

**M 375 56415 First Homework Due Feb. 4th**

1. Construct a binary linear code of length 5, dimension 2 and minimal distance 3. Show that there does not exist a binary linear code of length 5, dimension 3 and minimal distance 3. Show also that there does not exist a binary linear code of length 5, dimension 2 and minimal distance 4.

2. Magnetic tape for data storage in computers are usually recorded with nine heads, one on top of the other. That is, the data is stored in a matrix  $(b_{ij})$ ,  $i = 1, \dots, 9$ ,  $j = 1, \dots, n$ ,  $b_{ij} \in \mathbf{F}_2$ , for some  $n > 1$ . For ease of machine reading it is required that there is an "on" bit in every column, show that this is guaranteed by requiring that  $\sum_{i=1}^9 b_{ij} = 1$  for all  $j$ . Show also that this permits the correction of errors resulting of defects in one of the heads (a common occurrence). In order to correct other kinds of errors one can require also that  $\sum_{j=1}^n b_{ij} = n \pmod{2}$ , for every  $i$ . To analyze this code we may switch zeros and ones (ie set  $c_{ij} = b_{ij} + 1$  for all  $i, j$ ). The equations become then  $\sum_{i=1}^9 c_{ij} = \sum_{j=1}^n c_{ij} = 0$ . Compute the dimension and minimal distance of this code.

3. Let  $C$  be a code with  $w(C) = 2m$ , we know that  $C$  corrects  $m - 1$  errors. Prove that  $C$  detects  $m$  errors, that is, if we receive a message, we can detect whether it has at most  $m$  errors and if it has at most  $m$  errors we can tell how many errors it has.

4. Consider the code in  $\mathbf{F}_2^6$  consisting of vectors of the form  $(a, b, c, a + b, a + c, b + c)$  for all  $(a, b, c)$  in  $\mathbf{F}_2^3$ . For each of the received words below, decode, assuming at most one error, or show that at least two errors occurred.  $(1, 1, 1, 1, 0, 0)$ ,  $(1, 1, 0, 1, 0, 1)$ ,  $(1, 0, 1, 1, 1, 0)$

5. What is the minimal distance of the code

$$C = \{(1, 1, 1, 1, 0), (1, 0, 1, 1, 1), (1, 1, 1, 0, 1)\} \subset \mathbf{F}_2^5?$$

6. Does there exist a code in  $\mathbf{F}_2^4$  with three codewords and minimal distance 3? Justify.