

## M 375 – Homework 9

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### 4.4.1

Transpose  $H$  without the last three columns (identity matrix) to get the generator matrix  $A$ .

The generator matrix  $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$  computes the check digits. Since this is a  $4 \times 3$  matrix, the

message must be split into two parts  $m_1 = 0000$  and  $m_2 = 1101$ . For each part the check digits are computed by  $A \cdot m_i$ . The message  $m$  is transmitted as

$$m = (0000 \ 000 \ 1101 \ 010).$$

### 4.4.2

The message is  $m = (0000 \ 111 \ 0001 \ 110)$ . First, check parity by  $H \cdot m_i$ :

$$H \cdot m_1 = (111)$$

$$H \cdot m_2 = (101)$$

Both parts contain an error, because the parity check should result in (000). The location of the error is given by the result of the parity check, i.e. the parity check result matches a row in  $A$ . For the first part of  $m$  this is row one in  $A$ , for the second part it is row three. Therefore the error occurred at the first digit of the first part of  $m$  and in the third digit of the second part of  $m$ .

The corrected codeword is  $m = (1000 \ 0011)$ .

### 4.4.3

Length of the code  $n = 7$ , number of linearly independent columns (minimal distance)  $d = 3$ , number of codewords  $M = 2^{n-d} = 2^4 = 16$ .

### 4.5.1

To compute the corrector for each syndrome, look which column(s) of  $H$  one has to combine to generate the syndrome. Note: This choice might not be unique.

Syndrome	Corrector
000	0000000
001	0000001
010	0000010
011	0010000
100	0000100
101	1000000
110	0000110
111	0001100

- (a)  $x = 1111111$ ,  $Hx^t = 111$ , decode  $x$  as  $x' = 1110011$
- (b)  $x = 1101011$ ,  $Hx^t = 000$ , decode  $x$  as  $x' = 1101011$
- (c)  $x = 0110111$ ,  $Hx^t = 001$ , decode  $x$  as  $x' = 0110110$
- (d)  $x = 0111000$ ,  $Hx^t = 101$ , decode  $x$  as  $x' = 1111000$