1. Let R be a Noetherian domain, $P \subset R, P \neq (0)$ a prime ideal and $a \in R$ such that $P \subseteq (a) \subseteq R$. Prove that $P = (a)$ or $R = (a)$. (Moral: hypersurfaces have codimension one).

2. Let R be the polynomial ring $k[x]$ where k is field and $S = R[u]/(1 - xu)$ (the smallest extension of R where x has an inverse). Show that S is a PID (hint: If $I \subset S$ is an ideal, consider $I \cap R$). Conclude that $T = S[y, z]$ is a UFD (where y, z are new variables). Let V be the smallest subring of T containing $x, y, z, yz/x$. Show that y is irreducible in V (hint: show it is irreducible in T first). Show that V is isomorphic to $k[x_1, x_2, x_3, x_4]/(x_1x_2 - x_3x_4)$ and that V is not a UFD.