Valuation domains and local domains

Let V be a domain, F its field of fractions. We call V a valuation domain if, for every $x \in F$ either $x \in V$ or $x^{-1} \in V$. Let M be a maximal ideal of V and I an arbitrary ideal. If $x \in I$ and $x \notin M$, let $y \in M, y \neq 0$. Then $x/y \notin V$, for otherwise $x = (x/y)y$ would be in M, so $y/x \in V$ and $y = (y/x)x \in I$. So we conclude that $M \subset I$ and since M is maximal $M = I$ and x does not exist. Thus $I \subset M$ and, in particular, M is the unique maximal ideal of V.

A domain with an unique maximal ideal is called a local domain. The ring of power series in two variables over a field is a local domain which is not a valuation domain. $R = k[[x, y]]$ has a unique maximal ideal (x, y) ($R/(x, y)$ is isomorphic to k by $f \mapsto f(0, 0)$ so (x, y) is maximal and every $f \in R, f(0, 0) \neq 0$ is invertible). But R is not a valuation domain, since neither x/y nor y/x are in R.