M362k   Test # 2

Directions: Indicate all answers on the answer sheet provided. Please enter your name and student # where requested. An assertion is to be interpreted as a true-false question and answered "A" if true, "E" if false. Each question has only one correct answer.

1. Suppose X is a *discrete* random variable which takes on the values $-2,\ -1,\ 0,\ 1,\ 2$ with equal probability. What is $P(X < 0)$?
(A) 1/5 (B) 2/5 (C) 3/5 (D) 2/3 (E) None of these

2. Suppose X is the same as in problem **1.** Let $Y = X^2$. What is $P(Y \leq 1)$?
(A) 1/5 (B) 2/5 (C) 3/5 (D) 4/5 (E) None of these

3. Suppose we have a circular dartboard of radius 3 divided into three sections: a central disk of radius 1, and two concentric rings of radial width 1, labeled as in the following sketch:

(Region 1 is a disk, regions 2 and 3 are *rings.*)
Suppose a dart is thrown randomly at this board (with certainty that the board is hit somewhere; "randomly" means that the the probability of hitting some subregion is proportional to the area of the subregion.
Let X denote the number of the region hit; so X is a DVR with values in $\{1, 2, 3\}$. Which of the following is a correct description of the distribution of X?
(A) $P(X = 1) = 1/9,\ P(X = 2) = 1/3,\ P(X = 3) = 5/9$
(B) $P(X = i) = 1/3, \quad i = 1, .., 3$
(C) $P(X = 1) = 1/9,\ P(X = 2) = 4/9,\ P(X = 3) = 4/9$
(D) $P(X = 1) = 1/9,\ P(X = 2) = 2/9,\ P(X = 3) = 1/3$
(E) None of these

4. For the same dartboard described in problem **3.**, let Y be the random variable equal to the distance from the center of the point hit. Y is a continuous random variable. What is its *distribution function* $F(y) = P(Y \leq y)$?
(A) $F(y) = 2y, \ \ 0 \leq y \leq 3$ (B) $F(y) = \pi y^2, \ \ 0 \leq y \leq 3$ (C) $F(y) = 2y/9, \ \ 0 \leq y \leq 3$
(D) $F(y) = y^2/9, \ \ 0 \leq y \leq 3$ (E) None of these

5. Suppose X is a continuous random variable with the *pdf* $f(x) = 2x/3$ on $1 \leq x \leq 2$. (Note that $\int_1^2 (2x/3)\, dx = 1$.)
Let $Y = \ln X$. What is the *pdf* $g(y)$ of Y?
(A) $g(y) = 2\ln y/3$ on $[0, \ln 2]$ (B) $g(y) = 2e^y/3$ on $[0, \ln 2]$
(C) $g(y) = 2e^{2y}/3$ on $[0, \ln 2]$ (D) $g(y) = y^2/3$ on $[0, \ln 2]$ (E) None of these

6. Let X be the random variable described in problem **5.** Which of the following random variables is *uniformly distributed*? (This means the distribution is a *constant*.)
(A) $2X/3$ (B) $X^2/3$ (C) 3X/2 (D) $\sqrt{3X}$ (E) None of these

7. If X is a random variable which is *uniformly distributed* on the interval $[a, b]$, (which means that its *pdf* is the function $1_{[a,b]}/(b-a)$),
then $Y = 2X$ is also a *uniformly distributed* random variable (on *some* interval.)

8. If X is a random variable which is *uniformly distributed* on the interval $[a, b]$ then $Y = X^2$ is also a *uniformly distributed* random variable (on *some* interval.)

9. Suppose we throw a dart at the unit disk D in $\mathbb{R}^2$ (*i.e.* the set $x^2 + y^2 \leq 1$) in such a way that if $\mathbf{W} = (X, Y)$ is the point hit, then the *distribution* of $\mathbf{W}$ (*i.e.* the *joint distribution* of (X, Y)) has *pdf* $f(x, y) = C \cdot |x|$ where C is a constant. What is C?
(*Hint*: $\int\int_D f(x, y)\, dxdy = 1$.)
(A) 3/2 (B) 4/3 (C) 3/4 (D) 3 (E) None of these

10. In the situation described in problem **9.**, what is the (marginal) *pdf* of the random variable X?
(A) $C \cdot \int_{-1}^{1} |x|\, dx$ (B) $C \cdot \int_{-1}^{1} |x|\, dy$ (C) $C \cdot \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} |x|\, dx$ (D) $C \cdot \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} |x|\, dy$
(E) None of these

11. A segment of length L (which we may consider to be the interval $[0, L]$ in $\mathbb{R}$) is broken at a *random point*. Consider the random variable $S =$ *length of the shorter piece*. It is easy to show (and you may assume this) that the *pdf* of S is $f(s) = 2/L$ on the interval $0 \le s \le L/2$. What is the *expected length of the shorter piece*, $E(S)$?
(A) $L/4$ (B) $L/2$ (C) $L^2/4$ (D) 1 (E) None of these

12. In the situation described in problem **11.**, what is *expected length of the square of the shorter piece*, $E(S^2)$?
(**Note**: You don't have to compute the distribution of S^2 to compute $E(S^2)$.)
(A) $L^3/24$ (B) $L^2/12$ (C) $L^2/16$ (D) 1 (E) None of these

13. Suppose a fair coin ($P(H) = 1/2$) is tossed n times. Let X be the random variable equal to *the number of H occuring*. What is $E(X)$?
(A) $2n$ (B) $\sqrt{2n}$ (C) $1/2$ (D) $n/2$ (E) None of these

14. Suppose we toss a fair coin ($P(H) = 1/2$) repeatedly. Let Y be the randon variable equal to *the number of tosses till k H occur*. What is $E(Y)$?
(*Hint*: You could let $Y_1 =$ *the number of tosses till 1 H occurs*, $Y_2 =$ *the number of additional tosses till the next H occurs*, etc.)
(A) $k/2$ (B) $2k$ (C) $k^2/2$ (D) $k(k-1)/2$ (E) None of these

15. Again, suppose we toss a fair coin ($P(H) = 1/2$) repeatedly. Let Z be the randon variable equal to *the number of tosses till both H and T occur*. What is $E(Z)$?
(A) 3 (B) 4 (C) 5 (D) 2 (E) None of these

ANSWERS:
B C A D C B A E C D
A B D B A