M408C Sample Test 2 Name: ____________________

Directions: Indicate all answers on the answer sheet provided. Please enter your name and student # where requested. An assertion is to be interpreted as a true-false question and answered "A" if true, "E" if false. Each question has only one correct answer.
Turn in legible scratchwork for possible partial credit. (Even for a True-False question, you can give a reason or counterexample.)

1. Suppose y is a function of x satisfying $y^3 - y - x = 6$, and that $y(0) = 2$. What is $y'(0)$? (*Hint*: Differentiate implicitly and solve for y'.)
(A) 1/2 (B) 1/13 (C) 1/11 (D) 7/11 (E) None of these

2. If $f(x)$ is differentiable everywhere, and $f(1) = 3$, $f(2) = 1$, then by the Mean Value Theorem, there is a c with $1 < c < 2$ and $f'(c) =$
(A) 2 (B) -2 (C) 1/2 (D) -1/2 (E) None of these

3. Suppose $f(x)$ is a function defined whenever $x \neq 0$. Suppose that $f'(x) < 0$ for all x where f is defined. Then we can conclude that $f(1) < f(-1)$.

4. Suppose $f(x) = 3\sqrt{x} - \frac{x}{2}$, for $x > 0$. Then at $x = 1$, f
(*Note*: $f'(x) = \frac{3}{2\sqrt{x}} - \frac{1}{2}$.)
(A) is decreasing (B) is increasing (C) has local max (D) has local min
(E) None of these

5. For the same $f(x)$ as in **4**, on what interval is f *decreasing*?
(A) $(9, \infty)$ (B) $(0, 9)$ (C) $(\sqrt{3}, \infty)$ (D) $(0, \sqrt{3})$ (E) None of these

6. For the same $f(x)$ as in **4**, at $x = 9$, f
(A) is decreasing (B) is increasing (C) has local max (D) has local min
(E) None of these

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7. Consider the function $f(x) = x^3 - 3x - 1$ defined for all real x. $f(x)$ has a **local maximum** at some point $x = c$. What is $f(c)$? (*Note**** We are asking for $f(c)$, not c!)
(A) -1 (B) 0 (C) -3 (D) 1 (E) None of these

8. Now consider the function $f(x) = x^3 - 3x - 1$ defined on $[-10, 10]$. What is the **absolute maximum** of this function?
(A) 1 (B) 10 (C) 969 (D) Doesn't exist (E) None of these

9. Suppose now that $g(x) = x^4 - x^3$ defined for all real x. Which of the following is true for this function?
(A) $g(3/4)$ is a local max (B) $g(3/4)$ is a local min (C) $g(0)$ is a local max
(D) $g(0)$ is a local min (E) None of these

10. Suppose $h(x)$ is a function defined for all real x, and
$h'(0) = 0$, $h''(0) = 0$, $h'(x) > 0$ for $x < 0$, $h'(x) < 0$ for $x > 0$. Which of the following is true for this function?
(A) $h(0)$ is a local max (B) $h(0)$ is a local min (C) $(0, h(0))$ is an inflection point
(D) $h(0)$ is not a local extreme (E) None of these

11. Suppose $f(x)$ is defined for all real x. Which of the following implies that $f(0)$ is a local maximum?
(A) $\{f'0) = 0,\ f''(0) = 0\}$ (B) $\{f'0) = 0,\ f(0) = 1\}$ (C) $\{f'0) = 0,\ f''(0) > 0\}$
(D) $\{f'0) = 0,\ f''(0) < 0\}$ (E) None of these

12. Suppose $g(x)$ is defined and differentiable for all real x, and $g'(x) = 0$ only for $x = 0$. Suppose also that $g(0) = 1$, and $\lim_{x\to\pm\infty} g(x) = 0$. Which of the following is a correct conclusion?
(A) The absolute min of g is 1. (B) The absolute max of g is 1.
(C) g has a local max at $x = 0$, but it may not be the absolute max.
(D) g has an absolute max, and an absolute min. (E) None of these

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13. What is the absolute maximum of $f(x) = \frac{x}{1+x^2}$ (defined for all x)?
(*Note****: We want the **value** of f, not the x where the max occurs.)
(A) 0 (B) 1 (C) 1/2 (D) 2 (E) None of these

14. What is the maximum possible **area** of a rectangle whose total perimeter is L?
(*Note**** We are asking for the **area**, not some length of a side.)
(A) $L^2/16$ (B) $L^2/4$ (C) $L^2/8$ (D) L^2 (E) None of these

15. Consider the function $g(x) = x^3 - x^2$ defned for all x. On which of the following intervals is $g(x)$ concave down?
(A) $(-\infty, \frac{2}{3})$ (B) $(\frac{1}{3}, \infty)$ (C) $(0, \infty)$ (D) $(-\infty, \frac{1}{3})$ (E) None of these

16. Which of the following functions $h(x)$ has an inflection point at $(0, 0)$?
(A) $h(x) = x^2$ (B) $h(x) = x^3 - x^2$ (C) $h(x) = x^5$ (D) $h(x) = x^5 + x^4$
(E) None of these

17. Consider the function $f(x) = x$ on $[0, 1]$, and the partition $P = \{0, \frac{1}{2}, 1\}$ of $[0, 1]$. What is the *lower sum* $L_f(P)$ corresponding to this data?
(Recall that this is a sum of the form $\sum_{i=1}^{2} m_i(\Delta x)_i$ where m_i denotes a minimum value.)
(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{3}{4}$ (D) 1 (E) None of these

18. With the same function and partition as in the previous problem, what is the *Riemann Sum* obtained if we use the *midpoint* of each of the 2 subintervals of the partition as the points x_i^* $(i = 1, 2)$ where f is evaluated?
(Recall that this is a sum of the form $\sum_{i=1}^{2} f(x_i^*)(\Delta x)_i$ where x_i^* are points in the subintervals; here we assume these are midpoints.)
(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{3}{4}$ (D) 1 (E) None of these

19. What is $\int_1^4 \sqrt{x}\, dx$?
(A) 1 (B) 14/3 (C) -1/4 (D) 16/3 (E) None of these

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20. What is $\int_0^{\pi/4} \sec^2(x)\,dx$?

(A) 1/3 (B) 0 (C) $\frac{\sec^3(x)}{3}\Big|_0^{\pi/4}$ (D) 1 (E) None of these

21. Suppose $f(x) = \int_1^x \frac{dt}{1+t^4}$. What is $f'(2)$?

(A) $\pi/5$ (B) -16/17 (C) 1/17 (D) 1 (E) None of these

22. Suppose $f(x) = \int_1^{x^2} \frac{dt}{1+t^4}$. What is $f'(x)$?

(A) $\frac{2x}{1+x^8}$ (B) $\frac{2x}{1+x^4}$ (C) $\frac{1}{1+x^8}$ (D) $\frac{1}{1+x^6}$ (E) None of these

Answers:

C B E B A C D C B A

D B C A D C B A B D

C A