Applying Theorem (N18) 2 in a Proof

Problem: Suppose n and k are positive integers greater than 1 such that $18k = 33n$.

Prove that $11 \mid k$ and $3 \mid n$.

Proof: It is given that $18k = 33n$.

$33 = 11 \times 3$, so $11 \mid 33$ and $33 \mid 33n$.

So, by Theorem 4.3.3 (Divisibility is Transitive), $11 \mid 33n$.

$\therefore$ By substitution $11 \mid 18k$.

By Theorem (N18) 2, [with $a = 18$ and $b = k$], since 11 is a prime number, $11 \mid 18$ or $11 \mid k$.

Since $\frac{18}{11} = 1 + \frac{7}{11}$, $1 < \frac{18}{11} < 2$. $\therefore$ $\frac{18}{11}$ is not an integer.

$11 \nmid 18$. $\quad \therefore$ $11 \mid k$, by Elimination. $\quad [11 \mid k]$

Now, $3 \times 6k = 18k = 33n = 3 \times 11n$.

By Dividing by 3, we conclude that $6k = 11n$.

Since $6 = 3 \times 2$, $3 \mid 6$ and $6 \mid 6k$.

$\therefore$ $3 \mid 6k$, by Theorem 4.3.3.

$\therefore$ $3 \mid 11n$, and, since 3 is prime, $3 \mid 11$ or $3 \mid n$, by Theorem (N18) 2.

Since $\frac{11}{3} = 3 + \frac{2}{3}$, $3 < \frac{11}{3} < 4$. $\therefore$ $\frac{11}{3}$ is not an integer.

$\therefore$ $3 \nmid 11$. Therefore, by Elimination, $3 \mid n$. $\quad [3 \mid n]$

$\therefore$ $11 \mid k$ and $3 \mid n$, by Conjunction.

QED