<u>In-the-Book Definitions ( I )</u>

The "In-the-Book Definitions" are the definitions given in the book of terms, some of which are well known and understood already with perhaps other simpler and equivalent definitions. However, the "In-the-Book Definitions" are the definitions which are necessary for using these terms in a proof. Although the simpler definitions learned in the past are useful for understanding the objects defined, these simpler definitions are often very difficult to use when writing a proof of a theorem which involves these objects.

Since the "In-the-Book Definitions" are the definitions to be used in proofs, it is important to <u>memorize</u> these definitions word for word. One advantage to memorizing these definitions is that, when one of these definitions is applied within a proof, the exact wording of the "In-the-Book Definition" can be used directly in the wording of the proof.

<u>Definition:</u> Let n be an integer .

Integer n is an <u>even integer</u> $\Leftrightarrow$ there exists an integer k such that $n = 2k$ .

Integer n is an <u>odd integer</u> $\Leftrightarrow$ there exists an integer k such that $n = 2k + 1$ .

It can be proved that every integer is either even or odd and that no integer is both even and odd. The even-ness or odd-ness of an integer is called its <u>parity</u> .
The terms <u>even number</u> and <u>odd number</u> mean "even integer" and "odd integer," respectively.

<u>Definition:</u> Let n be an integer .

Integer n is <u>prime</u> $\Leftrightarrow$

( $n > 1$ ) AND

( For all positive integers r , s , IF $n = rs$ , THEN ( $r = 1$ OR $s = 1$ ) ) .

Integer n is <u>composite</u> $\Leftrightarrow$

( $n > 1$ ) AND ( there exist integers r and s such that

$1 < r < n$ and $1 < s < n$ and $n = rs$ ) .

The number 1 is neither prime nor composite.

It can be proved that every integer $> 1$ (which is greater than 1) is either prime or composite and no integer is both prime and composite.

The terms <u>prime number</u> and <u>composite number</u> mean "prime integer" and "composite integer," respectively.

Definition: Let r be a real number.

Real number r is *rational* $\Leftrightarrow$

There exist integers a and b such that $r = a / b$ and $b \neq 0$.

Real number r is *irrational* $\Leftrightarrow$ r is not rational.

The terms *rational number* and *irrational number* mean "rational real number" and "irrational real number" respectively.

In mathematics, the phrase "There exist objects x and y such that ... " always includes the possibility that x and y represent the same object, that is that $x = y$.
To require that x and y are not identical, the phrase "There exist *distinct* objects ... ".

Thus, the integers a and b in the definition above cannot be assumed to be distinct, that is, it is possible that they are equal (i.e., they represent the same number).
Therefore, $r = 1$ is a rational number.

Definition: Let n and d be integers.

- n *is divisible by* d
- n *is a multiple of* d
- d *divides* n (written $d \mid n$)
- d *is a divisor of* n
- d *is a factor of* n

$\Leftrightarrow$ There exists an integer k such that $n = d \cdot k$.

The integer 0 is divisible by every integer since $0 = 0 \cdot k$, for every integer k.

The integer 0 is a factor of no integer other than 0 itself since $n = 0 \cdot k$ implies $n = 0$.

It can be proved that for all *positive* values of d and n, $d \mid n \leftrightarrow n \div d$ has remainder 0.

Take care that "$d \mid n$" is not accidentally written "$d / n$" because "$d / n$" represents a number whereas "$d \mid n$" represents a relationship between integers. In fact, it can be proved that $d \mid n \Leftrightarrow n / d$ is an integer, and in this case, $k = n / d$.