

PROOF THAT THE SUM OF TWO RATIONAL NUMBERS
IS A RATIONAL NUMBER.

TO PROVE: The sum of any two rational numbers is a rational number.

$$[\forall r, s \in \mathbb{Q}, (r+s) \in \mathbb{Q}]$$

Proof: Let r and s be rational numbers.

[We need to show (NTS) that $(r+s)$ is rational.]

By definition of "rational," there exist integers

a, b, c, d , with $b \neq 0$ and $d \neq 0$, such that

$$r = \frac{a}{b} \text{ and } s = \frac{c}{d}.$$

$$\text{Since } \frac{b}{b} = \frac{d}{d} = 1, r = \frac{a}{b} = \frac{ad}{bd} \text{ and } s = \frac{c}{d} = \frac{bc}{bd}$$

by Rules of Algebra.

$$\text{So, } r+s = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd} \text{ by substitution and Rules of Algebra.}$$

Let $m = ad+bc$ and let $n = bd$, which are integers

since products and sums of integers are integers.

Also, $n \neq 0$ since $n = bd$ and $b \neq 0$ and $d \neq 0$ and by the ZERO PRODUCT PROPERTY.

$$\therefore (r+s) = \frac{m}{n} \text{ by substitution.}$$

Also m and n are integers and $n \neq 0$.

$\therefore (r+s)$ is rational, by Definition of "Rational."

\therefore The sum of two rational numbers is rational, by Direct Proof.

Q.E.D.