EXAMPLES OF PROOFS THAT Prove A FUNCTION is One-to-one

EXAMPLE 1: Define $T: \mathbb{Z} \to \mathbb{Z}$ as follows:

For all $n \in \mathbb{Z}$, $T(n) = 7n + 6$.

TO Prove: T is a one-to-one function.

Proof: [Recall: $f: X \to Y$ is one-to-one $\iff \forall u, v \in X$, If $f(u) = f(v)$, Then $u = v$.]

Let u and v be integers (in the Domain of T).

Suppose $T(u) = T(v)$.

$\therefore$ By definition of T, $T(u) = 7u + 6$ and $T(v) = 7v + 6$.

$\therefore 7u + 6 = 7v + 6$ by substitution.

$\therefore 7u = 7v$ [subtracting 6]

$\therefore u = v$ [dividing by 7]

$\therefore$ For all $u, v \in \mathbb{Z}$, if $T(u) = T(v)$, THEN $u = v$, by Direct Proof.

$\therefore T$ is one-to-one, by def'n of "one-to-one."

QED

EXAMPLE 2 : Let $h: \mathbb{R} \rightarrow \mathbb{R}^+$ be defined by the rule $h(x) = e^{(\frac{1}{3}x)}$, for all $x \in \mathbb{R}$.

To Prove: Function h is one-to-one.

Proof: Let s and t be real numbers.

[You could also say "Let $s \in \mathbb{R}$ and $t \in \mathbb{R}$ be given".]

Suppose $h(s) = h(t)$.

$\therefore$ By definition of function h,

$h(s) = e^{(\frac{1}{3}s)}$ and $h(t) = e^{(\frac{1}{3}t)}$.

$\therefore e^{(\frac{1}{3}s)} = e^{(\frac{1}{3}t)}$ by substitution

$\therefore \ln(e^{(\frac{1}{3}s)}) = \ln(e^{(\frac{1}{3}t)})$ by function properties.

$\therefore \frac{1}{3}(s) = \frac{1}{3}(t)$ by properties of e^x, and $\ln x$.

$\therefore s = t$. [multiply by 3]

$\therefore$ For all $s, t \in \mathbb{R}$, if $h(s) = h(t)$, then $s = t$, by Direct Proof.

$\therefore$ Function h is a one-to-one function, by definition of "one-to-one".

QED