EXAMPLE OF A PROOF THAT PROVES A FUNCTION IS ONTO.

EXAMPLE:

Let $h : \mathbb{R} \to \mathbb{R}^+$ be defined by the rule

$h(x) = e^{(\frac{1}{3}x)}$, for all $x \in \mathbb{R}$.

To Prove: FUNCTION h is ONTO.

Proof: [Recall: Function $f : X \to Y$ is ONTO $\iff \forall y_0 \in Y, \exists x_0 \in X$ such that $f(x_0) = y_0$.]

[WORKSPACE:
Given $y_0 \in Y = \mathbb{R}^+$, we seek $x_0 \in \mathbb{R}$ with $h(x_0) = y_0$;
That is, $e^{(\frac{1}{3}x_0)} = y_0$,
That is $\frac{1}{3}x_0 = \ln(y_0)$
That is $x_0 = 3\ln(y_0)$
So $h(3\ln(y_0)) = y_0$]

Let $y_0 \in \mathbb{R}^+$ be given.

Let $x_0 = 3\ln(y_0)$, which is defined because $y_0 > 0$.

$h(x_0) = e^{(\frac{1}{3}x_0)}$ by def'n of h,

$= e^{(\frac{1}{3}(3\ln y_0))}$ by substitution,

$= e^{\ln(y_0)}$

$= y_0$ by properties of e^x and $\ln(x)$.

$\therefore h(x_0) = y_0$.

$\therefore$ For all $y \in \mathbb{R}^+$, there exists an element $x \in \mathbb{R}$ such that $h(x) = y$, by Direct Proof.

$\therefore h$ is onto, by def'n of "ONTO".

QED.