

Solution of the Exercise Deduction of the DEDUCTION FORMAS HANDOUT

List of Given Premises :

- a) $(\sim p \vee q) \rightarrow r$
 b) $s \vee \sim q$
 c) $\sim t$
 d) $p \rightarrow t$
 e) $(\sim p \wedge r) \rightarrow \sim s$

TO Deduce : $\sim q$

Problem Analysis : (THIS IS NOT THE SOLUTION YET!)

Final Argument:

$$\frac{s \vee \sim q}{\sim s} \therefore \sim q \text{ by Elimination}$$

[Thus, in an earlier argument, we will need to deduce the truth of $\sim s$]

Earlier Argument:

$$\frac{(\sim p \wedge r) \rightarrow \sim s}{(\sim p \wedge r)} \therefore \sim s \text{ by Modus Ponens}$$

[So, we will need to have previously deduced $(\sim p \wedge r)$]

Even earlier Argument: $\sim p$

$$\frac{\sim p}{r} \therefore \sim p \wedge r \text{ by Conjunction}$$

[We need to have deduced earlier $\sim p$ and r individually]

Deducing $\sim p$:

$$\frac{p \rightarrow t}{\sim t} \therefore \sim p \text{ by Modus Tollens}$$

Deducing r takes several steps:

Deducing r :

$$\frac{\sim p}{\therefore \sim p \vee q} \text{ by Generalization, followed by:}$$

$$\frac{(\sim p \vee q) \rightarrow r}{\sim p \vee q} \therefore r$$

We can now present the deduction by giving these arguments in REVERSE ORDER !!

DEDUCTION (THIS IS THE SOLUTION!)

3

| <u>STATEMENT</u> | <u>JUSTIFICATION</u> |
|--|--|
| $ \begin{array}{l} 1. \quad p \rightarrow t \\ \quad \quad \sim t \\ \hline \therefore \sim p \end{array} $ | $ \begin{array}{l} \text{by Premise (d)} \\ \text{by Premise (c)} \\ \hline \text{by Modus Tollens} \end{array} $ |
| $ \begin{array}{l} 2. \quad \sim p \\ \hline \therefore \sim p \vee q \end{array} $ | $ \begin{array}{l} \text{by Argument (1)} \\ \hline \text{by Generalization} \end{array} $ |
| $ \begin{array}{l} 3. \quad (\sim p \vee q) \rightarrow r \\ \quad \quad (\sim p \vee q) \\ \hline \therefore r \end{array} $ | $ \begin{array}{l} \text{by Premise (a)} \\ \text{by Argument (2)} \\ \hline \text{by Modus Ponens} \end{array} $ |
| $ \begin{array}{l} 4. \quad \sim p \\ \quad \quad r \\ \hline \therefore \sim p \wedge r \end{array} $ | $ \begin{array}{l} \text{by Argument (1)} \\ \text{by Argument (3)} \\ \hline \text{by Conjunction} \end{array} $ |
| $ \begin{array}{l} 5. \quad (\sim p \wedge r) \rightarrow \sim s \\ \quad \quad (\sim p \wedge r) \\ \hline \therefore \sim s \end{array} $ | $ \begin{array}{l} \text{by Premise (e)} \\ \text{by Argument (4)} \\ \hline \text{by Modus Ponens} \end{array} $ |
| $ \begin{array}{l} 6. \quad s \vee \sim q \\ \quad \quad \sim s \\ \hline \therefore \sim q \end{array} $ | $ \begin{array}{l} \text{by Premise (b)} \\ \text{by Argument (5)} \\ \hline \text{by ELIMINATION} \end{array} $ |

3

Note: ALTHOUGH This is unnecessarily complicated,
you could replace the first "modus tollens"

$$\text{argument } \left(\begin{array}{l} p \rightarrow t \quad \text{by premise (d)} \\ \sim t \quad \text{by premise (c)} \\ \hline \therefore \sim p \quad \text{by Modus Tollens} \end{array} \right)$$

with the following "modus Ponens" Argument
by replacing the first premise " $p \rightarrow t$ "
with its equivalent contrapositive " $\sim t \rightarrow \sim p$ ":

| <u>STATEMENT</u> | <u>JUSTIFICATION</u> |
|--------------------------------|--|
| 1. $\sim t \rightarrow \sim p$ | by Premise (d) Equivalence using the Contrapositive |
| $\sim t$ | by Premise (c) |
| $\therefore \sim p$ | by Modus Ponens |