AN EXAMPLE OF REWORDING A QUANTIFIED STATEMENT

In the following, f represents a real-valued function of a real variable, That is, f is a function, $f : \mathbb{R} \to \mathbb{R}$.

PROBLEM: Reword the given statement S to an equivalent statement which does not use the word "sufficient."

S: "Being continuous is not a sufficient condition for a function f to be differentiable."

It is not at first obvious what should be done here. It seems to be a negation of another statement.

It would help to formulate a wording of the statement that S is the negation of.

So, $S \equiv \sim T$, where T is as follows:

T: "Being continuous is a sufficient condition for a function f to be differentiable."

Now, the reference to "a function f" means the same as "any function f," that is, T is a UNIVERSAL STATEMENT. Also, the use of the term "sufficient condition" suggests that this statement T could equivalently be rephrased as a universal conditional (IF-THEN) statement.

AN EXAMPLE of Rewording ... (cont.)

$T \equiv$ "For all functions f, if f is continuous, then f is differentiable."

Now, the original problem was to reword $S \equiv \sim T$.

To help in formulating the negation of T, it helps to determine the symbolic representation of T first, and then apply the rules to form the symbolic representation of its negation.

Let D be the collection of all functions $f : \mathbb{R} \to \mathbb{R}$.

In Symbols: T: $\forall f \in D$, If f is continuous, Then f is differentiable.

$S \equiv \sim T \equiv$ $\exists f \in D$ such that $\sim$("If f is continuous, Then f is differentiable.")

The negation of "$p \to q$" is "$p \wedge \sim q$".

$S \equiv \sim T \equiv$ "$\exists f \in D$ such that "f is continuous AND f is not differentiable."

One correct rewording of S is the following:

"There exists a function f such that f is continuous AND f is not differentiable."