

TWO WAYS TO PROVE A UNIVERSAL STATEMENT USING PROOF-BY-CONTRADICTION

To Prove: $\forall x \in D, P(x)$.

[METHOD 1: WRITING A
DIRECT PROOF WITH AN
ARGUMENT INSIDE THAT
USES PROOF-BY-CONTRADICTION]

Proof: let x be any object in D .
[N.T.S. $P(x)$]

Suppose, by way of contradiction,
that $\neg P(x)$.

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$\therefore$  "THIS THING" is true,  
WHICH CONTRADICTS  
"THAT THING".

$\therefore P(x)$ , by proof-by-contradiction.

$\therefore$  For all  $x \in D$ ,  $P(x)$  by Direct  
Proof.

QED.

[METHOD 2: WRITING ONE  
BIG PROOF-BY-CONTRADICTION.]

Proof: Suppose, by way of  
contradiction, that there  
exists an object  $x$  in  $D$   
such that  $\neg P(x)$ .

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"THIS THING" is true,
which contradicts
"THAT THING".

\therefore For all $x \in D$, $P(x)$, by proof-
by-contradiction.

QED.