A THEOREM AND THE PARITY COROLLARY

THEOREM: Every integer n can be written as $n = 2k$ or as $n = 2k+1$, for some integer k.

[$\forall n \in \mathbb{Z}$, $\exists k \in \mathbb{Z}$ such that $n = 2k$ or $n = 2k+1$.]

Proof: Let n be any integer. Let $d = 2$.
[We apply the Q-R THEOREM to n and d.]

By the Quotient-Remainder Theorem, there exist unique integers q and r such that $n = 2q + r$ and $0 \le r < 2$.

$\therefore$ $r = 0$ or $r = 1$. [These are the cases to consider.]

CASE 1: ($r = 0$)

Suppose $r = 0$. Then, $n = 2q + 0 = 2q$.

$\therefore n = 2q$. $\therefore n = 2q$ or $n = 2q+1$, by generalization.

$\therefore$ There exists an integer k [here, $k = q$] such that $n = 2k$ or $n = 2k+1$, in CASE 1. [END of CASE 1]

CASE 2: ($r = 1$)

Suppose $r = 1$. Then, $n = 2q + 1$.

$\therefore n = 2q$ or $n = 2q + 1$, by generalization.

$\therefore$ There exists an integer k [here, $k = q$] such that $n = 2k$ or $n = 2k+1$, in CASE 2. [END of CASE 2].

$\therefore$ In General, there exists an integer k such that $n = 2k$ or $n = 2k+1$.

QED by Direct Proof.

THE PARITY COROLLARY:

For every integer n, n is even or n is odd.

Proof: Exercise, using the theorem above.