A Solution for Sec. 4.5, #13, which uses Theorem (N1B) 3

<u>To Prove</u>: For any integer n, n^2-2 is not divisible by 4.

<u>Proof</u>: [Proof-by-Contradiction]

Suppose, by way of contradiction, that there exists an integer n such that $4 \mid (n^2-2)$.

Therefore, by def'n of "divides", there exists an integer k such that $n^2-2 = 4k$.

$\therefore n^2 = 4k+2$

$= 2(2k+1)$

$\therefore n^2 = 2t$, where $t = 2k+1$, and t is an integer.

$\therefore 2 \mid n^2$, by def'n of "divides".

Since 2 is a prime number, $2 \mid n$, by Theorem (N1B) 3.

$\therefore n = 2l$, for some integer l, by definition of "divides".

$\therefore n^2 = (2l)^2$, by substitution,

$\therefore n^2 = 4l^2$, and l^2 is an integer.

$\therefore 4 \mid n^2$, by definition of "divides".

Since $4 \mid n^2$ and $4 \mid (n^2-2)$, $4 \mid [n^2-(n^2-2)]$, by Exercise #16 of Section 4.3 in the textbook.

Since $[n^2-(n^2-2)] = 2$, $4 \mid 2$, by Substitution.

$\therefore$ By Theorem 4.3.1, $4 \leq 2$, which contradicts the fact that $4 > 2$.

$\therefore$ For any integer n, n^2-2 is not divisible by 4, by proof-by-contradiction.

Q.E.D.