A Solution For Sec 4.5, #13, that uses DIVISION INTO CASES, But not Theorem (NIB) 3.

To Prove: For any integer n, n^2-2 is not divisible by 4.

Proof: Let n be any integer.

By the Parity Corollary, n is even or n is odd.

CASE 1 (n is even):

Suppose n is even.

Then, there exists an integer k such that $n=2k$.

Suppose, by way of contradiction, that n^2-2 is divisible by 4.

Then $n^2-2=4t$, for some integer t.

$\therefore$ $(2k)^2-2=4t$, by substitution.

$\therefore$ $4k^2-2=4t$, and so, $4k^2-4t=2$.

$\therefore$ $2=4(k^2-t)$, and k^2-t is an integer.

$\therefore$ $4|2$

By Theorem 4.3.1, $4 \le 2$, which contradicts the fact that $4>2$.

$\therefore$ n^2-2 is not divisible by 4, in the case that n is even, by proof-by-contradiction.

[END of CASE 1]

[The proof continues with CASE 2 on the next page] [solution for Sec 4.5; #13 (continued)]

CASE 2 (n is odd):

Suppose n is odd.

Then, there exists an integer k such that
$$n = 2k+1.$$

Suppose, by way of contradiction, that n^2-2 is divisible by 4.

Then $n^2-2 = 4t$ for some integer t.

$\therefore$ $(2k+1)^2-2 = 4t$, by substitution.

$\therefore$ By rules of algebra, $(4k^2+4k+1)-2 = 4t$.

$\therefore$ $4k^2+4k-1 = 4t$, and so, $4k^2+4k-4t = 1$.

$\therefore$ $4(k^2+k-t) = 1$, and (k^2+k-t) is an integer.

$\therefore$ $4 \mid 1$.

$\therefore$ By Theorem 4.3.1, $4 \leq 1$, which contradicts the fact that $4 > 1$.

$\therefore$ n^2-2 is not divisible by 4, in the case that n is odd, by proof-by-contradiction. [END of CASE 2].

$\therefore$ n^2-2 is not divisible by 4, in General!

$\therefore$ For any integer n, n^2-2 is not divisible by 4, by Direct Proof.

Q.E.D.