Two Proofs of the same statement with structures worth comparing

To Prove: For every integer n, $5n+3$ is not divisible by 5.

Proof 1:

Let n be any integer

Suppose, by way of contradiction, that $5 \mid (5n+3)$.

$\therefore 5n+3 = 5k$, for some integer k.

$\therefore 3 = 5k - 5n$

$\therefore 3 = 5(k-n)$, and $k-n$ is an integer.

$\therefore 5 \mid 3$,

$\therefore$ By Theorem 4.3.1, $5 \leq 3$, which contradicts the fact that $5 > 3$.

$\therefore 5 \nmid (5n+3)$, by proof-by-contradiction

$\therefore$ For every integer n, $5n+3$ is not divisible by 5, by Direct Proof.

QED.

Proof 2

Suppose, BWOC, that there exists an integer N, such that $5 \mid (5N+3)$.

$\therefore$ For some integer k,

$5N+3 = 5k$.

$\therefore 3 = 5k - 5N$

$\therefore 3 = 5(k-N)$, and $k-N$ is an integer.

$\therefore 5 \mid 3$.

$\therefore$ By Theorem 4.3.1, $5 \leq 3$, which contradicts the fact that $5 > 3$.

$\therefore$ For every integer n, $5n+3$ is not divisible by 5, by proof-by-contradiction.

QED.