HW #11B, PART I SOLUTIONS

M325K
SPRING 2024

SECTION 8.5:

#3. $21 = 3\times 7$, so $7 \mid 21$. $\therefore \gcd(7, 21) = 7$.

#4. USING THE EUCLIDEAN ALGORITHM:

$$\begin{array}{r} 1 \\ 48\overline{)54} \\ -48 \\ \hline 6 \end{array} \qquad \begin{array}{r} 8 \\ 6\overline{)48} \\ -48 \\ \hline 0 \end{array} \qquad \therefore \gcd(48, 54) = 6.$$

Using prime factorizations:

$48 = 6\times 8 = 3\times 16 = 2^4\times 3^1$

$54 = 6\times 9 = 2\times 27 = 2^1\times 3^3$.

$\therefore$ The $\gcd(48, 54) = 2^1\cdot 3^1 = 6$

#5. FIND $\gcd(1188, 385)$.

$$\begin{array}{r} 3 \\ 385\overline{)1188} \\ -1155 \\ \hline 33 \end{array} \qquad \begin{array}{r} 11 \\ 33\overline{)385} \\ -33 \\ \hline 55 \\ -33 \\ \hline 22 \end{array} \qquad \begin{array}{r} 1 \\ 22\overline{)33} \\ -22 \\ \hline 11 \end{array} \qquad \begin{array}{r} 2 \\ 11\overline{)22} \\ -22 \\ \hline 0 \end{array}$$

$\therefore \gcd(1188, 385) = 11$

#6 FIND $\gcd(509, 1177)$

$$\begin{array}{r} 2 \\ 509\overline{)1177} \\ -1018 \\ \hline 159 \end{array} \qquad \begin{array}{r} 3 \\ 159\overline{)509} \\ -477 \\ \hline 32 \end{array} \qquad \begin{array}{r} 4 \\ 32\overline{)159} \\ -128 \\ \hline 31 \end{array} \qquad \begin{array}{r} 1 \\ 31\overline{)32} \\ -31 \\ \hline 1 \end{array} \qquad \begin{array}{r} 31 \\ 1\overline{)31} \\ -31 \\ \hline 0 \end{array}$$

$\therefore \gcd(509, 1177) = 1$.

(cont.)

Sec. 8.5

#7 Find gcd(832, 10933)

$$\begin{array}{r@{\,}l} & 13 \\ 832 & \overline{)10933} \\ & \underline{832} \\ & 2613 \\ & \underline{2496} \\ & 117 \end{array} \qquad \begin{array}{r@{\,}l} & 7 \\ 117 & \overline{)832} \\ & \underline{819} \\ & 13 \end{array}$$

$$\begin{array}{r@{\,}l} & 9 \\ 13 & \overline{)117} \\ & \underline{117} \\ & 0 \end{array} \qquad \therefore \gcd(832, 10933) = 13.$$

#8. Find gcd(4131, 2431)

$$\begin{array}{r@{\,}l} & 1 \\ 2431 & \overline{)4131} \\ & \underline{2431} \\ & 1700 \end{array} \qquad \begin{array}{r@{\,}l} & 1 \\ 1700 & \overline{)2431} \\ & \underline{1700} \\ & 731 \end{array}$$

$$\begin{array}{r@{\,}l} & 2 \\ 731 & \overline{)1700} \\ & \underline{1462} \\ & 238 \end{array} \qquad \begin{array}{r@{\,}l} & 3 \\ 238 & \overline{)731} \\ & \underline{714} \\ & 17 \end{array} \qquad \begin{array}{r@{\,}l} & 14 \\ 17 & \overline{)238} \\ & \underline{17} \\ & 68 \\ & \underline{68} \\ & 0 \end{array}$$

$\gcd(4131, 2431) = 17.$

(CONT.)

Section 8.5, #9.

To Prove: For all positive integers a and b,
$a | b$ if, and only if, $\gcd(a,b) = a$.

Proof: Let a and b be positive integers.

Let $d = \gcd(a,b)$. Then, by definition of "gcd", $d|a$ and $d|b$, and d is the greatest of all such common divisors of a and b.

[First prove: If $a|b$, then $\gcd(a,b) = a$]

Suppose that a/b.

Since $a \cdot 1 = a$, $a|a$. $\therefore$ $a|a$ and $a|b$.

$\therefore$ a is a common divisor of a and b.

Since d is the greatest common divisor of a and b, and a is a common divisor of a and b, we conclude that $a \leq d$.

Recall that $d|a$. [and $d|b$, too, since d is a common divisor]

$\therefore$ By Theorem 4.3.1, $d \leq a$.

Since $d \leq a$ and $a \leq d$, we conclude that $d = a$.

$\therefore$ $\gcd(a,b) = a$, by substitution.

$\therefore$ If $a|b$, then $\gcd(a,b) = a$.

[Next prove: If $\gcd(a,b) = a$, then a/b. Recall: $d = \gcd(a,b)$.]

Suppose $d = a$. Then $a|b$, by substitution, since $d|b$.

$\therefore$ If $\gcd(a,b) = a$, then $a|b$.

QED, by direct proof.