HW #12 — M325K SPRING 2024

Sec 7.1 Solutions

#7 (a) $F(\{1,3,4\}) = 1$ since 3 is odd.

(b) $F(\emptyset) = 0$ since 0 is even.

(c) $F(\{2,3\}) = 0$ since 2 is even

(d) $F(\{2,3,4,5\}) = 0$ since 4 is even.

#12 $J_5 = \{0,1,2,3,4\}$

Define $G: J_5 \times J_5 \to J_5 \times J_5$ as follows:
For all $(a,b) \in J_5 \times J_5$,

$$G(a,b) = ((2a+1) \bmod 5,\ (3b-2) \bmod 5)$$

(a) $G(4,4) = (9 \bmod 5,\ 10 \bmod 5) = (4,0)$

(b) $G(2,1) = (5 \bmod 5,\ 1 \bmod 5) = (0,1)$

(c) $G(3,2) = (7 \bmod 5,\ 4 \bmod 5) = (2,4)$

(d) ——— $G(1,5)$ is not defined because $(1,5) \notin J_5 \times J_5$.

Sec. 7.1, #14. $J_5 = \{0, 1, 2, 3, 4\}$, $h: J_5 \to J_5$ and $k: J_5 \to J_5$ such that $h(x) = ((x+3)^3 \bmod 5)$ and $k(x) = ((x^3 + 4x^2 + 2x + 2) \bmod 5)$, for all x in J_5.

The function values of h:

x	$h(x) = ((x+3)^3 \bmod 5)$
0	2
1	4
2	0
3	1
4	3

Calculations:
$(0+3)^3 = 27 \equiv 2 \pmod 5 \Rightarrow h(0) = 2$
$(1+3)^3 = 64 \equiv 4 \pmod 5 \Rightarrow h(1) = 4$
$(2+3)^3 = 5^3 \equiv 0^3 = 0 \pmod 5 \Rightarrow h(2) = 0$
$(3+3)^3 = 6^3 \equiv 1^3 = 1 \pmod 5 \Rightarrow h(3) = 1$
$(4+3)^3 = 7^3 \equiv 2^3 = 8 \equiv 3 \pmod 5 \Rightarrow h(4) = 3$

The function values of k:

x	$k(x) = (x^3+4x^2+2x+2) \bmod 5$
0	2
1	4
2	0
3	1
4	3

Calculations:

$0^3 + 4 \times 0^2 + 2 \times 0 + 2 = 2$
$\Rightarrow k(0) = (2 \bmod 5) = 2$

$1^3 + 4 \times 1^2 + 2 \times 1 + 2 = 9 \equiv 4 \pmod 5$
$\Rightarrow k(1) = 4$

$2^3 + 4 \times 2^2 + 2 \times 2 + 2 = 30 \equiv 0 \pmod 5$

$3^3 + 4 \times 3^2 + 2 \times 3 + 2 = 27 + 36 + 8$
$\equiv (2 + 1 + 3) \equiv 6 \equiv 1 \pmod 5$

Sec 7.1, # 14 (cont.)

<u>More Calculations:</u>

$$4^3 + 4 \times 4^2 + 2 \times 4 + 2$$
$$= 64 + 64 + 8 + 2$$
$$\equiv (4 + 4 + 3 + 2) \pmod 5$$
$$\equiv 13 \pmod 5$$
$$\equiv 3 \pmod 5$$

$\Rightarrow k(x) = 3$

Since Domain h = Domain k (Both $= J_5$)
and Co-Domain h = Co-Domain k (Both $= J_5$)

and, For all $x \in J_5$, $h(x) = k(x)$,

Functions h and k are equal functions, $\underline{\underline{h = k}}$.