M 325K Homework #1, Part II Solutions

From the handout DEFINING VARIABLES IN A PROOF

Exercise #1:

All the variables are adequately defined when they are used. The variable **z** (defined in the statement "Let **z** = **x** + y,") inherits its declaration as a particular entity-type from the declarations of **x** and **y**. If we view 4 and 6 as being integers, then **z** inherits a declaration as an integer. If we view 4 and 6 as being real numbers, then **z** inherits a declaration as a real number. This ambiguity (though not serious here) is another reason why inherited declarations are to be avoided.

Exercise #2:

The variable **y** is not adequately defined. In its first use it is not declared as representing a particular entity-type nor is it defined in terms of other adequately defined variables so it does not inherit a declaration. Since **z** is defined in terms of an inadequately defined variable, **z** is not adequately defined.

Since **w** is defined in terms of the inadequately defined variable **x**, **w** is not adequately defined either, even though **x** is adequately defined.

The situation is remedied by inserting statements which adequately define the variable **y**. This will result in **z** being adequately defined, which in turn will result in **w** being adequately define.

"Let **x** be an integer less than 5.
Let y be any integer. (Added statement)
Let **z** = **y** + 3.
Let **w** = **x** + **z**.
Let **q** = **x** + 3.
Therefore, **x** > 20."

(Note that the final conclusion of this argument is false so this argument is invalid. The truth or falsity of the conclusion is not an issue here. The only issue in these exercises is whether the variables are adequately defined whenever they are used.

Exercise #3:

In the statement "Let **w** = **x** + **z**," the variables **x** and **z** are both not adequately defined. They were locally adequately defined during their use in the previous If-Then statements, but they lost their definitions when they were used without redefinition.

Since **w** is defined in terms of at least one inadequately defined variable (in this case two inadequately defined variables), the variable **w** is also inadequately defined.

The situation is remedied by inserting a statement defining the variables **x** and **z**. This first correction shown is a correction in poor style because the locally defined variables are used later as the same symbol defined globally. This is formally correct IF THE GLOBAL DEFINITION DOES NOT PRECEDE THE USE OF THE SAME VARIABLE IN A LOCAL REDEFINITION. To avoid extreme confusion, the global definitions of **x** and **z** come after their use as locally defined variables in the If-Then statements. It is extremely confusing if a globally defined variable is used later with a local redefinition and this should be avoided at all costs.

Exercise #3 (Continued):

Correction in poor style:

"Let **y** be a positive integer.
If **x** is 4, then **(x + y) > (y + 3).**
If **z** is 6, then **(z + y) > (y + 5).**
Let x and z be any two (not necessarily distinct) positive integers.
Let **w = x + z.**
Therefore, **w** = 10."

Correction in unacceptably poor style (with globally defined variables used later with local redefinition):

"Let **y** be a positive integer.
Let **x** and **z** be any two (not necessarily distinct) positive integers.
If **x** is 4, then **(x + y) > (y + 3).**
If **z** is 6, then **(z + y) > (y + 5).**
Let **w = x + z.**
Therefore, **w** = 10."

<u>Correction in good style</u>:

"Let **y** be a positive integer.
If ***r*** is 4, then **(*r* + *y*) > (y + 3).**
If ***r*** is 6, then **(*r* + *y*) > (y + 5).**
Let x and z be any two (not necessarily distinct) positive integers.
Let **w = x + z.**
Therefore, **w** = 10."

(In none of these arguments, as written here, is the conclusion justified. For example **x** could be 17 and **z** could be 6, in which case **w** would be 23 and not 10.)

Exercise #4:

In the statement "Let **z** = **x + y**," the variables **x** is not adequately defined so **z** is not adequately defined there either Variables **k** and **m** are adequately defined. In the statement "Let **n** = **d + c**," the variables **d** is not adequately defined. Variable **d** was locally adequately defined during its use in the previous If-Then statement, but it lost its definition when it was used without redefinition. Consequently, variable **n** is also not adequately defined. (Note that the conclusion is justified in this argument.)

The situation is remedied by inserting a statement defining the variables **x** and **d**. For good style, the use of variable **d** in the If-Then statement has been replaced by the use of variable **g** because variable **d** is used later as a globally defined variable.

Correction:

"Suppose **a = 1, b = 3**, and **c** is a negative integer.
If ***g*** is an integer less than **c**, then ***g*** is negative.
Let **y** be any real number greater than 1,000.
*Let x be an integer and let **d** be an integer.*
Let **z** = **x + y.**
Let **k** = **a + c.**
Let **m** = **b + c.**
Let **n** = **d + c.**
Therefore, **k+m** = **2c + 4.**"

Exercise #5: It looks at first glance as if all the variables used are adequately defined, but in fact variable **z** is not adequately defined. The domain of the variable is to some extent limited to numbers between 500 and 1,000, but it does not say what *kind* of numbers they are:

integers between 500 and 1,000? ; rational numbers between 500 and 1,000? ;

real numbers between 500 and 1,000? ;

The variable **z** needs to be explicitly declared as representing a particular entity when it is first used.

Correction:

"Let **y** be any real number greater than 1,000.
Let **x** be any integer less than –1,000.
Let **z** *be any integer* which is greater than 500 but less than 1,000.
Therefore, $(\mathbf{y} + \mathbf{x} - \mathbf{z})$ is a negative number."

Exercise #6: Again, it looks at first glance as if all the variables used are adequately defined, but in fact variable **t** is not adequately defined. The domain of the variable is to some extent limited to numbers greater than 5, but it does not say what *kind* of number **t** is: integer?; rational number? real # ?

Since **x** and **y** are defined in terms of **t**, the variable **x** and **y** are also not adequately defined.

The variable **t** needs to be explicitly declared as representing a particular entity when it is first used.

"Let **t** *be any real number such that* $\mathbf{t} > 5$.
Suppose **y** is an integer such that $\mathbf{y} > \mathbf{t}$.
Let $\mathbf{x} = \mathbf{t} + 10$.
Therefore, $\mathbf{x} > \mathbf{y}$."