# Homework #2, Part IV, Solutions

**A.** Here $n = 7$.

Assignment: Determine all integers $x$ such that

$$50 \le x \le 90 \quad \text{and} \quad x \equiv 68 \pmod{7}.$$

Sol'n: Since $68 - 68 = 0 = 7 \times 0$ and $0$ is an integer,

$$68 \equiv 68 \pmod{7}.$$

$\therefore$ By a Theorem in a posted handout, the results of the following calculations are also congruent to 68 modulo 7:

$$\begin{array}{r} 68 \\ +7 \\ \hline 75 \end{array}, \quad \begin{array}{r} 75 \\ +7 \\ \hline 82 \end{array}, \quad \begin{array}{r} 82 \\ +7 \\ \hline 89 \end{array}, \quad \begin{array}{r} 89 \\ +7 \\ \hline 96 \end{array}, \quad \text{etc.} \qquad \text{and}$$

$$\begin{array}{r} 68 \\ -7 \\ \hline 61 \end{array}, \quad \begin{array}{r} 61 \\ -7 \\ \hline 54 \end{array}, \quad \begin{array}{r} 54 \\ -7 \\ \hline 47 \end{array}, \quad \text{etc.}$$

Since $50 \le x \le 90$, the values of $x$ such that $x \equiv 68 \pmod{7}$ are

$$x = 54, 61, 68, 75, 82, 89.$$

B. Assignment: Determine all integers $n$ such that

$$41 \equiv 17 \pmod{n}.$$

Solution: Since $n$ is a modulus for the "congruence modulo $n$" relationship, $n$ is a positive integer.

Now, $41 - 17 = 24$, so

$41 \equiv 17 \pmod{n}$ if and only if

$41 - 17 = n \cdot k$ for some integer $k$,

which is true if and only if
$24 = n \cdot k$ for some integer $k$.

Thus $41 \equiv 17 \pmod{n}$ if and only if $n$ is a positive integer factor (also called a positive divisor) of 24. The factorizations of 24 as a product of two positive integers are:

$$24 = 1\times 24 = 2\times 12 = 3\times 8 = 4\times 6 = 6\times 4 = 8\times 3 = 12\times 2 = 24\times 1.$$

So the positive integer factors of 24 are
$n = 1, 2, 3, 4, 6, 8, 12, 24$.

Thus, $47 \equiv 17 \pmod{n}$ if and only if $n$ is one of the numbers: $1, 2, 3, 4, 6, 8, 12, 24$