SECTION 3.1 — SPRING

HW #2, Solutions, M325K, 2024

Sec 3.1

2a This is true by definition.

2b This is false. Positive numbers are greater than 0.

2c This is False. A counterexample is $r = -1$. When $r = -1$, $-r = +1 > 0$.

2d This is false. A counterexample is $r = \frac{1}{2}$, which is not an integer.

3a $P(2) =$ "$2 > \frac{1}{2}$" This is TRUE.

$P(\frac{1}{2}) =$ "$\frac{1}{2} > 1/\frac{1}{2}$" This is False.

$P(-1) =$ "$-1 > 1/(-1)$" This is False.

$P(-\frac{1}{2}) =$ "$-\frac{1}{2} > 1/-\frac{1}{2}$" This is True.

$P(-8) =$ "$-8 > 1/(-8)$" This is False.

3b. Suppose $x > 0$. Then $x > \frac{1}{x} \iff x^2 > 1$
$\iff x > 1$

Suppose $x < 0$,
Then $x > \frac{1}{x} \iff x^2 < 1$
$\iff -1 < x < 0$

The Truth set of $P(x)$ is
$\{x \in \mathbb{R} \mid -1 < x < 0 \text{ OR } x > 1\}$
$= (-1, 0) \cup (1, \infty)$

3c If Domain $= \mathbb{R}^+$, The Truth set is $(1, \infty)$
$= \{x \in \mathbb{R} \mid x > 1\}$.

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(NOT ASSIGNED) **#5** $Q(x,y) =$ "If $x<y$, Then $x^2<y^2$."

a) $Q(-2,1) =$ "If $-2<1$, Then $4<1$"
This is False because "$-2<1$" is True and "$4<1$" is False, and in this situation, the IF-THEN STATEMENT is FALSE.

b) For $x = -3$ and $y = 2$,
$Q(-3,2) =$ "If $-3<2$, Then $9<4$"
which is a false statement.

c) If $x=3$ and $y=8$, Then
$3<8$ is True and $9<64$ is true too,
so $Q(3,8) =$ "If $3<8$, THEN $9<64$" is TRUE.

d) For $x=4$ and $y=5$, $Q(4,5) =$
"If $4<5$, Then $16<25$" is True.

6. a $R(25,10) =$ "If 25 is a factor of 100, Then 25 is a factor of 10."
is false because 25 is a factor of 100 but 25 is not a factor of 10.

b. For $m=9$ and $n=6$,
$R(9,6) =$ "If 9 is a factor of 36, Then 9 is a factor of 6" is a false IF-THEN.

6c and 6d were not assigned

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#6c $R(5,10)$ = "If 5 is a factor of 100, Then 5 is a factor of 10"

Is True because 5 is a factor of 100 and 5 is a factor of 10 and in this situation, The IF-THEN statement is TRUE.

#6d For $m = 3$ and $n = 15$,

$R(3,15)$ = "If 3 is a factor of 15, then 3 is a factor of 225."

$15 = 3 \times 5$ and

$225 = 3 \times 75$ so $R(3,15)$ is TRUE.

#7 a) For $\frac{6}{d}$ to be an integer, with $d \in \mathbb{Z}$

d can be 6, 3, 2, 1, −1, −2, −3 or −6.

The Truth set of the predicate is

$\{-6, -3, -2, -1, 1, 2, 3, 6\}$

7b For $\frac{6}{d}$ to be an integer with $d \in \mathbb{Z}^+$,

d can be 6, 3, 2 or 1.

The Truth set is $\{1, 2, 3, 6\}$

7c When the Domain of x is $\mathbb{R}$, The Truth set is $\{x \in \mathbb{R} \mid -2 \leq x \leq -1 \text{ OR } 1 \leq x \leq 2\}$.

$= [-2, -1] \cup [1, 2]$.

#7d, When Domain of x is $\mathbb{R}$, The TRUTH SET is

$\{-2, -1, 1, 2\}$

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#8 : $B(x) =$ "$-10 < x < 10$"

a) $\{-9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

b) $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

c) $\{-8, -6, -4, -2, 0, 2, 4, 6, 8\}$

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#10. Interpret the phrase
"(a-1)/a is not an integer" to mean

"(a-1)/a does not represent an integer".

Even with this interpretation, the universal statement in the problem is false. There is exactly one counterexample. When $a=1$, $a-1=0$, so $(a-1)/a=0$ and 0 is an integer. So, $a=1$ is the counterexample.

#16)

(#16 was not assigned)

b) $\forall$ real number x, x is positive, negative or zero.

c) $\forall$ irrational number, x is not an integer.

f) $\forall$ real number x, $x^2 \neq -1$.

~~#28 a) There exists a prime which is not odd.~~
~~Some primes are not odd. True, p = 2~~
~~c) There is an integer that is an odd perfect square.~~ ~~$9 = 3^2$ is odd.~~
~~Some perfect squares are odd.~~
Some ~~odd~~ numbers are ~~perfect squares~~.

Section 3.1

#18 a) $\exists s \in D$ such that $E(s) \wedge M(s)$,
Also OK is: $\exists s \in D$ such that $E(s)$ AND $M(s)$.

b) $\forall s \in D,\ C(s) \to E(s)$
Also OK is: $\forall s \in D$, if $C(s)$, then $E(s)$

c) Here are four Equivalent solutions:

$\forall s \in D,\ C(s) \to \sim E(s)$

$\forall s \in D,\ E(s) \to \sim C(s)$

$\forall s \in D,\ \sim(C(s) \wedge E(s))$

$\forall s \in D,\ \sim C(s) \vee \sim E(s)$.

These are all Equivalent.

d) $\exists s \in D$ such that $C(s)$ AND $E(s)$

e)
($\exists s \in D$ such that $C(s)$ AND $E(s)$) AND
($\exists s \in D$ such that $C(s)$ AND $\sim E(s)$)

#30 a) There exists a prime which is not odd.
Some primes are not odd. TRUE, $p=2$

c) There is an integer that is an odd perfect square.
Some perfect squares are odd.
Some odd numbers are perfect squares.
TRUE: $9 = 3^2$ is an odd number.

Sec 3.1 #32

a) This is true. Any real number greater than 2 is also greater than 1.

b) This is true; The square of any number greater than 2 is greater than 4.

c) This is false.

$x^2 > 4 \Rightarrow x > 2$ means

$\forall x \in \mathbb{R}$, if $x^2 > 4$, then $x > 2$.

A counter-example is $x = -3$.

$x^2 = (-3)^2 = 9$, so $x^2 > 4$, but $x \leq 2$ since $-3 \leq 2$ and so x is not greater than 2.

d) $x^2 > 4 \Leftrightarrow |x| > 2$ is true since

$x^2 > 4 \Leftrightarrow x < -2$ or $x > 2$ and

$|x| > 2 \Leftrightarrow x < -2$ or $x > 2$.