1

HW #3 SECTION 3.2 SOLUTIONS
M325K SPRING 2024

SEC. 3.2

#2 Negations of "All dogs are loyal":

c. Some dogs are disloyal.
f. There is a dog that is disloyal

#3: a) $\exists$ fish x such that x does not have gills.

b) $\exists$ computer c such that c does not have a cpu.

c) $\forall$ movies m, m is not over 6 hours long.

d) $\forall$ band b, b has not won at least 10 Grammy awards.

#9) The negation of the statement
"$\forall$ real numbers x, if $x > 3$, then $x^2 > 9$"

is the statement

"$\exists$ real number x such that $x > 3$ and $x^2 \leq 9$."

Sec 3.2

#13 The proposed negation is not a correct negation.

A correct negation is as follows:

There exists an integer n such that n^2 is even, but n is not even.

#17: The Negation:
$\exists$ integer d such that $6/d$ is an integer and $d \neq 3$.

#22: THE NEGATION:
There is an integer which is not odd, but its square is odd.

Also Ok: There is an integer whose square is odd, but it is not odd itself.

~~#34: ORIGINAL STATEMENT:~~
~~$\forall$ animals x, if x is a dog, then x has paws and x has a tail.~~

~~Contrapositive:~~
~~$\forall$ animals x, if x does not have paws or x does not have a tail, then x is not a dog.~~

Sec 3.2

#32 ORIGINAL STATEMENT:
"If the square of an integer is odd, then the integer is odd.

This is True.

The Converse:
"If an integer is odd, then its square is odd."
This is True.

The Inverse:
"If the square of an integer is not odd, then the integer is not odd."
This is true.

The Contraposition:
"If an integer is not odd, then its square is not odd."

This is True.

#42 If one obtains a Master's degree, then they passed a comprehensive exam.

Sec 3.2 :

#44. THE STATEMENT is The Negation of a conditional:

It is not the case that having a large income is a necessary condition for a person to be happy.

STATED EQuivalently:

It is not the case that, if one does not have a large income, then one is not a happy person. ≡ If you are happy, then you have a large income.

Since the negation of a conditional statement is an AND statement [$\sim(p \to q) \equiv p$ AND $\sim q$], This can be stated equivalently as:

There exists a person who is happy and does not have a large income.

Informally stated.

Some people without a large income are happy.

This is equivalent to the original statement because

$\sim(\forall x$, if $P(x)$, then $Q(x)) \equiv \exists x$ such that $P(x)$ AND $\sim Q(x)$.

Sec 3.2

#46: The statement is the negation of a conditional:
It is not the case that, if a function is a polynomial function, then it has a root which is a real number.

Because $\sim(\forall x, \text{ if } P(x), \text{ then } Q(x))$
$$\equiv$$
$\exists x$ such that $P(x)$ AND $\sim Q(x)$,

this can be stated equivalently as:

There exists a function f such that f is a polynomial function and f does not have a root which is a real number.

Informally: Some polynomial functions do not have a real root.