M325K SPRING 2024

HW #3, SECTION 4.1 Solutions,
PROBLEMS 5, (13), 19

#5

TO prove: There are distinct integers m and n such that $\frac{1}{m}+\frac{1}{n}$ is an integer.

Proof: Let $m=1$ and let $n=-1$.

$\frac{1}{m}+\frac{1}{n}=\frac{1}{1}+\frac{1}{-1}=1+(-1)=0$ and 0 is an integer and $1\neq -1$.

$\therefore$ There are distinct integers such that $\frac{1}{m}+\frac{1}{n}$ is an integer, by proof-~~by~~-example.

QED.

#9 TO Prove:

There is a perfect square that can be written as a sum of two other perfect squares.

[To Prove: $\exists n \in \mathbb{Z}$ such that (n is a perfect square) AND ($\exists a, b \in \mathbb{Z}$ such that ($n \neq a$ AND $n \neq b$ AND (a and b are perfect squares) AND $n^2 = a^2 + b^2$.]

Proof: Let $n=25$; let $a=9$; let $b=16$.

$25=5^2$; $9=3^2$; $16=4^2$. $\therefore$ n, a and b are perfect squares.

Sec 4.1, #9. (continued)

$\therefore n = a + b$ since $25 = 9 + 16$.
Also, a, b, and n are three distinct integers.
$\therefore$ There is a perfect square that can be written as the sum of two other perfect squares, by proof-by example. ∎

#13 To Disprove: For all integers m and n, if $2m+n$ is odd, then m and n are both odd.

We disprove this by proving its negation.

To Prove: The statement "For all integers m and n, if $2m+n$ is odd, then m and n are both odd" is FALSE.

Proof:
Let $m = 6$ and $n = 5$. Then, $2m + n = 17$.
Now, $17 = 2 \times 8 + 1$; so, by definition of "odd", 17 is odd. $\therefore$ $2m + n$ is odd, by substitution.
Now, $m = 6 = 3 \times 2$, so, by definition of "even", m is even.
$\therefore$ m is even or n is even, by Generalization.

Note: This statement is optional.

$\therefore$ There exist integers m and n such that $2m+n$ is odd and either m is even or n is even, by proof-by-example.

$\therefore$ The statement "For all integers m and n, if $2m+n$ is odd, then m and n are both odd" is proved FALSE by proof-by-counterexample. ∎

Sec. 4.1

#19

Part (a) $\forall$ integers m and n, if m is even and n is odd, then $m+n$ is odd.

$\forall$ even integers m and odd integers n, $m+n$ is odd.

If m is an even integer and n is an odd integer, then $m+n$ is odd.

Part (b)

(a) ... n is any odd integer.

(b) ... $m = 2r$ for some integer r,

(c) $m+n = 2r + (2s+1) = 2(r+s)+1$

(d) ..., and so $m+n$ is odd, by definition of "odd".