

HW 3A, PART A SOLUTIONS

M 325K

SOLUTIONS for The "Beginning of a Proof-by-Contradiction" Worksheet

Assignment #1:

To Prove: For every integer $n > 1$, $n < n^2 - 1$.

Proof: Suppose, by way of contradiction, that there exists an integer $n > 1$ such that

$$n \geq n^2 - 1. \quad \dots$$

Assignment #2:

To Prove: There exists an integer n such that $5n = 45$.

Proof: Suppose, by way of contradiction, that for every integer n , $5n \neq 45$

Assignment #3:

To Prove: For all integers $n \neq 0$, $|n| > n$ or $n > (1/2)n$.

Proof: Suppose, by way of contradiction, that there exists an integer $n \neq 0$

such that $|n| \leq n$ and $n \leq (1/2)n$

Assignment #4:

To Prove: For all real numbers r and s , if $0 < r < s$, then $r^4 < s^4$.

Proof: Suppose, by way of contradiction, that there exist real numbers r and s such that

$$0 < r < s \quad \text{and} \quad r^4 \geq s^4. \quad \dots$$

Assignment #5:

To Prove: For all real numbers r and s , if r is a rational number and s is not a rational number, then the sum $r + s$ is not a rational number.

Proof: Suppose, by way of contradiction, that there exist real numbers r and s such that

r is a rational number and s is not a rational number and the sum $r + s$ is a rational number.

. . . .