M325K SPRING 2024

HW #3A, Part C Solutions

Solution to the "Proving Equations and Inequalities" Worksheet:

Part I

To Prove: $\frac{\sqrt{2}+4}{7} < \frac{4}{5}$

WORK SPACE

The Proof Appears to the right. On this side is a Workspace in which calculations are made to discover the proof:

$$\frac{\sqrt{2}+4}{7} < \frac{4}{5}$$

(multiply by 7): $\sqrt{2}+4 < \frac{4}{5}\times 7 = \frac{28}{5}$

(multiply by 5:) $5\sqrt{2}+20 < 28$

(Subtract 20) $5\sqrt{2} < 8$

(divide by 5) $\sqrt{2} < \frac{8}{5}$

(Square both sides) $2 < \frac{64}{25}$

(multiply by 25) $50 < 64$ CHECK ✓

This technique in not valid to write directly in a proof. It is only for Proof discovery.

Proof:

$50 < 64$ by properties of Integers

$50 \times \frac{1}{25} = 64 \cdot \frac{1}{25}$ by Rules of Algebra (R.O.A.)

$\therefore 2 < \frac{64}{25} = \frac{8^2}{5^2}$

$\therefore \sqrt{2} < \frac{8}{5}$ since $y = \sqrt{x}$ is increasing on $[0,\infty)$

$\therefore 5\sqrt{2} < 8$ by R.O.A.

$\therefore 5\sqrt{2}+20 < 8+20 = 28$

$\therefore 5(\sqrt{2}+4) < 28$ by ROA.

$\therefore \sqrt{2}+4 < \frac{28}{5}$ by ROA

$\therefore (\sqrt{2}+4)\frac{1}{7} < \frac{28}{5}\cdot\frac{1}{7} = \frac{4\times 7}{5}\cdot\frac{1}{7}$

$\therefore \frac{\sqrt{2}+4}{7} < \frac{4}{5}$, by proof by direct verification.

QED

HW 3A, Part C Solutions (Continued)

Part II

(1) This proof is NOT Acceptable because the 3^{rd} statement, "$1+2+3+\cdots+k+(k+1) = \frac{(k+1)(k+2)}{2}$," is a conclusion that has not been proved using previous statements in the proof and previous theorems and the rules of logic.

(2) <u>To Prove</u>: For every integer $k \geq 3$, if $1+2+3+\cdots+k = \frac{k(k+1)}{2}$, then $1+2+3+\cdots+k+(k+1) = \frac{(k+1)(k+2)}{2}$.

<u>Proof</u>: Let k be any integer such that $k \geq 3$.
Suppose that $1+2+3+\cdots+k = \frac{k(k+1)}{2}$.

$\therefore$ $1+2+3+\cdots+k+(k+1) = (1+2+3+\cdots+k)+(k+1)$ by Rules of Algebra.

$\therefore$ $1+2+3+\cdots+k+(k+1) = \frac{k(k+1)}{2}+(k+1)$, by Substitution,

$= k\cdot\frac{(k+1)}{2} + 2\left(\frac{k+1}{2}\right)$, by R.O.A.,

$= (k+2)\left(\frac{k+1}{2}\right) = \frac{(k+1)(k+2)}{2}$, by ROA.

$\therefore$ $1+2+3+\cdots+k+(k+1) = \frac{(k+1)(k+2)}{2}$ by Transitivity of "=".

$\therefore$ For every integer k, $k \geq 3$, if $1+2+3+\cdots+k = \frac{k(k+1)}{2}$, then $1+2+3+\cdots+k+(k+1) = \frac{(k+1)(k+2)}{2}$, by Direct Proof.

QED