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M325K HW #4, Sec. 4.1 SOLUTIONS

Section 4.1, #28 SPRING 2024

(ASSIGNED IN PART A)

To Prove: For all integers n, if n is odd, then n^2 is odd.

Proof: Let n be any integer.

Suppose n is odd. [NTS: n^2 is odd.]

$\therefore$, by definition of "odd", there exists an integer k such that $n = 2k+1$.

$$\begin{aligned} \therefore n^2 &= (2k+1)^2 && \text{, by substitution,} \\ &= 4k^2+4k+1 \\ &= 2(2k^2+2k)+1 && \text{, by rules of algebra.} \end{aligned}$$

Let $t = 2k^2+2k$, which is an integer since sums and products of integers are integers.

$\therefore n^2 = 2t+1$, by substitution.

$\therefore n^2$ is odd, by definition of "odd."

$\therefore$ For all integers n, if n is odd, then n^2 is odd, by Direct Proof.

QED.

Sec. 4.1, #39.

In applying the definition of "even" to m, it was an error to use the variable "k" to represent the integer such that m equals 2 times it, because "k" had already been defined as the integer k such that $n = 2k+1$.

A different variable, such as l, should have been used, saying, for instance, "By definition of 'even', $m = 2l$ for some integer l."

#42. In applying the definition of "even" to n, it was an error to use the variable "k" to represent the integer such that n equals 2 times it, because "k" had already been defined as the integer k such that $m = 2k$.

A different variable, such as l, should have been used, saying, for instance, "By defintion of "even", $m = 2l$ for some integer l."

Sec 4.1 # 46

To Prove: The product of any even integer and any integer is even.

[Formal restatement: $\forall m, n \in \mathbb{Z}$, if m is even, then $m \cdot n$ is even.]

Proof Let m and n be any integers.

Suppose m is even. [NTS: $m \cdot n$ is even]
$\therefore$ By definition of "even", $m = 2k$ for some integer k.

$\therefore$ $m \cdot n = (2k)n$, by substitution,
$= 2(kn)$ by rules of algebra.

Let $t = kn$, and t is an integer since a product of integers is an integer.

$\therefore$ $mn = 2t$, by substitution.
$\therefore$ By definition of "even", mn is even.

$\therefore$ The product of any even integer and any integer is even, by DIRECT Proof.
QED.

Sec 4.1 ← NOT ASSIGNED.

#49 To Prove: The difference of any two odd integers is even.

[Formal restatement: $\forall m,n \in \mathbb{Z}^{ODD}$, $m-n$ is even]

Proof: Let m and n be any two odd integers.
[NTS: $m-n$ is even]

[One could also say: "Let m and n be any integers. Suppose m and n are both odd."]

By definition of "odd," there exist integers k and l such that $m=2k+1$ and $n=2l+1$.

$\therefore m-n = (2k+1)-(2l+1)$, by substitution

$= 2k+1-2l-1$

$= 2k-2l$

$= 2(k-l)$, all by rules of algebra.

Let $t=k-l$, which is an integer since the difference of integers is an integer.

$\therefore m-n = 2t$, by substitution.
$\therefore m-n$ is even, by definition of "even".

$\therefore$ The difference of any two odd integers is even, by DIRECT PROOF. Q.E.D.