

M325K HW #4, sec 4.2, solutions / SPRING 2024

#10 let m and n be integers such that $n \neq 0$.

$5m + 12n$ is an integer and $4n$ is an integer since sums and products of integers are integers.

$4n \neq 0$ by the zero product property.

\therefore By definition of "rational number",

$(5m + 12n) / (4n)$ is a rational number.

#17 To Prove: The difference of any two rational numbers is a rational number.

[Formal Restatement: $\forall r, s \in \mathbb{Q}, r - s \in \mathbb{Q}$.]

Proof: Let rational numbers r and s be given.

By definition of "rational", there exist integers $k, l, m,$ and n such that $r = k/l$ and $s = m/n$ and $l \neq 0$ and $n \neq 0$.

Now, $ln \neq 0$ by the Zero Product Property.

See 4.2, #17 (cont.)

$$\begin{aligned} m-n &= \left(\frac{k}{l} - \frac{m}{n} \right), \text{ by substitution,} \\ &= \frac{kn}{ln} - \frac{lm}{ln} \\ &= \frac{(kn-lm)}{ln}, \text{ by rules of algebra.} \end{aligned}$$

Since the sum, difference and product of integers are integers, $kn-lm$ and ln are integers. Also, recall that $ln \neq 0$.

∴ $m-n$ is a rational number, by definition of "rational number".

∴ The difference of any two rational numbers is a rational number, by DIRECT PROOF.

Q.E.D.

Sec 4.2, #38

Since "Fraction" another word for "rational number", when the writer of the proof uses, as a reason, for justifying a step, "The sum of two fractions is a fraction", the writer of the proof is assuming that the sum of two rational numbers is a rational number. This is what the writer is trying to prove.

Assuming the truth of the statement to be proven within the proof is an error. It is "Begging the Question."