M325K, SPRING 2024

HW #4, SECTION 4.3 SOLUTIONS

Sec. 4.3:

3.) Yes. $5 \mid 0$ because

$$0 = 5 \cdot 0$$

5.) Yes. $4 \mid 6m(2m+10)$

$$6m(2m+10) = 12m^2 + 60m$$
$$= 4(3m^2 + 15)$$

Let $t = 3m^2 + 15$, which is an integer

$\therefore$ $6m(2m+10) = 4t$ so

$$4 \mid 6m(2m+10).$$

THE ANSWER TO THE EXTRA QUESTION IS: $K = 3m^2 + 15$

12) Yes. Let k be an integer and

let $n = 4k+1$. Then,

$$n^2 - 1 = (4k+1)^2 - 1 = (16k^2 + 8k + 1) - 1$$
$$= 16k^2 + 8k$$
$$= 8(2k^2 + k).$$

Let $t = 2k^2 + k$, Then $n^2 - 1 = 8t$, so $8 \mid (n^2 - 1)$

THE ANSWER TO THE EXTRA QUESTION IS: $t = 2k^2 + k$.

Sec 4.3, #16.

TO Prove: For all integers a, b and c,

if $a|b$ and $a|c$, then $a|(b-c)$

<u>Proof</u>. Let a, b and c be any integers.
Suppose $a|b$ and $a|c$. [NTS: $a|(b-c)$]

By Def'n of "divides", there exist integers k and l such that
$b = ak$ and $c = al$.

Then $b - c = ak - al$ by substitution

$= a(k-l)$ by rules of algebra.

Let $t = k - l$, which is an integer.

$\therefore$ $b - c = at$, by substitution

$\therefore$ $a|(b-c)$ by definition of "divides."

$\therefore$ For all integers a, b, and c, if $a|b$ and $a|c$, then $a|(b-c)$, by Direct Proof.

QED

(NOT ASSIGNED) #27) This is false. As a counterexample,
let $a = 5$, $b = 6$ and $c = 4$.
then $b + c = 10$ and $5|10$, so $a|(b+c)$
but $5 \nmid 6$, so $a \nmid b$ and $5 \nmid 4$, so $a \nmid c$.

SEC. 4.3, #28 :

<u>TO PROVE</u>: The Statement "For all integers a, b, and c, if $a|b$, then $a|b$ or $a|c$" is false.

<u>Proof</u>: [We will exhibit a counterexample.]

Let $a = 10$, $b = 2$ and $c = 5$.

Then, $bc = 10$ and $10 = 10 \times 1$.

$\therefore$ $10|10$ and so $a|bc$, by substitution.

Now, $10 > 2$ and $10 > 5$.

$\therefore$ By Theorem 4.3.1 (and by Modus Tollens), $10 \nmid 2$ and $10 \nmid 5$.

$\therefore$ $a \nmid b$ and $a \nmid c$, by substitution.

$\therefore$ With $a = 10$, $b = 2$ and $c = 5$, $a|bc$ and $a \nmid b$ and $a \nmid c$.

$\therefore$ The statement "For all integers a, b and c, if $a|bc$, Then $a|b$ or $a|c$" is false by proof-by-Counter-example.

QED

Sec 4.3, #29 (NOT Assigned)

To Prove: For all integers a and b,
if $a \mid b$, then $a^2 \mid b^2$.

Proof: Let a and b be any integers such that $a \mid b$. [NTS: $a^2 \mid b^2$]

$\therefore$ By definition of "divides," $b = ak$, for some integer k.

Let $l = k^2$, which is an integer because the product of integers is an integer.

Now, $b^2 = (ak)^2$, by substitution,
$= (a^2)(k^2)$, by rules of algebra,
$= (a^2)\,l$, by substitution.

$\therefore$ $b^2 = a^2 l$ and l is an integer

$\therefore$ $b^2 \mid a^2$, by definition of "divides".

$\therefore$ For all integers a and b,
if $a \mid b$, then $a^2 \mid b^2$, by Direct Proof.
QED