HW #5A, PART II SOLUTIONS

① <u>TO PROVE</u>: FOR ALL INTEGERS $k \geq 3$,
if $2^0+2^1+2^2+\cdots+2^{k-1} = (2^k-1)$,

then $2^0+2^1+2^2+\cdots+2^k = (2^{(k+1)}-1)$.

<u>Proof</u>: Let k be any integer such that $k \geq 3$.
Suppose $2^0+2^1+2^2+\cdots+2^{k-1} = (2^k-1)$.

Now, $2^0+2^1+2^2+\cdots+2^{k-1}+2^k$

$$= (2^0+2^1+2^2+\cdots+2^{k-1})+2^k$$

$$= (2^k-1)+2^k, \text{ by substitution},$$

$$= (2^k+2^k)-1$$

$$= 2\cdot 2^k - 1.$$

$\therefore$ $2^0+2^1+2^2+\cdots+2^k = (2^{k+1}-1)$, by rules of algebra.

$\therefore$ For all integers $k \geq 3$,
if $2^0+2^1+\cdots+2^{k-1} = 2^k-1$,
then $2^0+2^1+\cdots+2^k = (2^{k+1}-1)$, by Direct Proof.

QED

HW #5A, Part II SOLUTIONS (continued)

(2) <u>To Prove</u>: FOR ALL INTEGERS $k \geq 1$,

$$\text{if } 10^0 + 10^1 + \cdots + 10^k < 10^{k+1},$$
$$\text{then } 10^0 + 10^1 + \cdots + 10^{k+1} < 10^{k+2}.$$

<u>Proof:</u> Let k be any integer such that $k \geq 1$.

(*) Suppose $10^0 + 10^1 + \cdots + 10^k < 10^{k+1}$.

[We need to show that $10^0 + 10^1 + \cdots + 10^k + 10^{k+1} < 10^{k+2}$.]

$$10^0 + 10^1 + \cdots + 10^{k+1} = (10^0 + 10^1 + \cdots + 10^k) + 10^{k+1}$$

Adding 10^{k+1} to both sides of the inequality in the supposition above (LINE (*)), we conclude that

$$(10^0 + 10^1 + \cdots + 10^k) + 10^{k+1} < 10^{k+1} + 10^{k+1}$$
$$\text{and } 10^{k+1} + 10^{k+1} = 2 \times 10^{k+1}$$

So, by substitution, $(10^0 + 10^1 + \cdots + 10^k) + 10^{k+1} < 2 \times 10^{k+1}$.

Since $2 < 10$, $2 \times 10^{k+1} < 10 \cdot 10^{k+1} = 10^{k+2}$.

$\therefore$ By Transitivity "<", $10^0 + 10^1 + \cdots + 10^{k+1} < 10^{k+2}$.

$\therefore$ By Direct Proof, for all integers $k \geq 1$,
if $10^0 + 10^1 + \cdots + 10^k < 10^{k+1}$,
Then $10^0 + 10^1 + 10^2 + \cdots + 10^{k+1} < 10^{k+2}$. Q.E.D.