HW #5B, PART II SOLUTIONS

To Prove: For all positive integers s and t such that $s > 1$ and $t > 1$,

If $75s = 35t$, THEN $7 | s$ and $5 | t$.

Proof: Let s and t be positive integers such that $s > 1$ and $t > 1$.

Suppose also that $75s = 35t$.

Then, [by dividing through by 5], $15s = 7t$. [EQUATION (*)]

$\therefore$ $7 | 15s$, by definition of "divides."

Since 7 is prime and $7 | (15s)$, by Theorem (N1B) 2,

$7 | 15$ or $7 | s$.

$\frac{15}{7} = 2\frac{1}{7}$. $\therefore$ $2 < \frac{15}{7} < 3$. $\therefore$ $\frac{15}{7}$ is not an integer. $\therefore$ $7 \nmid 15$.

$\therefore$ $7 | s$ by elimination.

From EQUATION (*) above, $7t = 15s$.

$\therefore$ $7t = 5(3s)$ and $3s$ is an integer.

$\therefore$ $5 | (7t)$ by definition of "divides."

Since 5 is prime and $5 | 7t$,

$5 | 7$ or $5 | t$, by Theorem (N1B) 2.

$\frac{7}{5} = 1\frac{2}{7}$. $\therefore$ $1 < \frac{7}{5} < 2$, $\therefore$ $\frac{7}{5}$ is not an integer. $\therefore$ $5 \nmid 7$.

$\therefore$ $5 | t$, by Elimination.

$\therefore$ $7 | s$ and $5 | t$, by conjunction.

QED, by Direct Proof.