The Basis Step Worksheet SOLUTIONS

M 325K

Problem 1: To Prove: For all integers $n \geq 2$, $5^n + 9 < 6^n$.

Proof:

[BASIS STEP] Let $n = 2$. $5^n + 9 = 5^2 + 9$, by substitution,

$5^n + 9 = 25 + 9 = 34$.

$6^n = 6^2$, by substitution.

$= 36$.

$34 < 36$.

$\therefore$ For $n = 2$, $5^n + 9 < 6^n$, by substitution. [END OF BASIS STEP]

Problem 2:

To Prove: For all integers $n \geq 2$,

$2^{2n} - 1$ is divisible by 3.

Proof:

[BASIS STEP]

Let $n = 2$.

$2^{2n} - 1 = 2^{2 \times 2} - 1$, by substitution

$= 2^4 - 1 = 16 - 1$

$\therefore 2^{2n} - 1 = 15$

$15 = 3 \times 5$.

$\therefore$ 15 is divisible by 3, by definition of "Divides".

$\therefore$ For $n = 2$, $2^n - 1$ is divisible by 3, by substitution.

[END OF BASIS STEP]

Problem 3:

To Prove: For all integers $n \geq 4$, $\sum_{j=0}^{n} 2^j = 2^{(n+1)} - 1$.

Proof: Let $n = 4$.

$$\sum_{j=0}^{n} 2^j = \sum_{j=0}^{4} 2^j \text{, by substitution,}$$

$$= 2^0 + 2^1 + 2^2 + 2^3 + 2^4$$

$$= 1 + 2 + 4 + 8 + 16$$

$$= 31$$

$$2^{n+1} - 1 = 2^{(4+1)} - 1 \text{, by substitution,}$$

$$= 2^5 - 1$$

$$= 32 - 1$$

$$= 31$$

$31 = 31$.

$\therefore$ For $n = 4$, $\sum_{j=0}^{n} 2^j = 2^n - 1$, by substitution.

[END OF BASIS STEP]