Homework #7B, Section 1.2 Solutions   M325K . SPRING 2024

SECTION 1.2

#7. a) $S = \{n \in \mathbb{Z} \mid n = (-1)^k,$ for some integer $k\}$

$$\boxed{S = \{-1, 1\}}$$

b) $T = \{m \in \mathbb{Z} \mid m = 1 + (-1)^i$ for some integer $i.\}$

$$\boxed{T = \{0, 1\}}$$

c) $U = \{r \in \mathbb{Z} \mid 2 \le r \le -2\}$

"$2 \le r \le -2$" means "$2 \le r$ AND $r \le -2$", a statement which is false for every $r \in \mathbb{Z}$.

$\boxed{U = \{\ \}}$ or "$U = \emptyset$" is ok.

d) $V = \{s \in \mathbb{Z} \mid s > 2 \text{ or } s < 3\}$

The statement "$s > 2$ or $s < 3$" is true for every integer s.

$\boxed{V = \{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}}$ or "$V = \mathbb{Z}$" is O.K.

Sec 1.2

#7 (e) $W = \{t \in \mathbb{Z} \mid 1 < t < -3\}$

$\boxed{W = \{\ \}}$ or "$W = \emptyset$" is OK

f) $X = \{u \in \mathbb{Z} \mid u \leq 4 \text{ or } u \geq 1\}$

$\boxed{X = \{\ldots -2, -1, 0, 1, 2, 3, \ldots\}}$ or "$X = \mathbb{Z}$" is ok.

#8 (a) No, $B \nsubseteq A$, because there is an element in B (for example, j) which is not an element of A.

(b) Yes, $C \subseteq A$, because every element of C is also an element of A.

(c) Yes, $C \subseteq C$, because every element of C is also an element of C.

(d) Yes, C is a proper subset of A because $C \subseteq A$ and $C \neq A$.

STATED IN ANOTHER WAY: every element of C is an element of A, but there exists an element of A which is not an element of A.

Sec 1.2, #9

a) Yes, $3 \in \{1,2,3\}$

b) No, $1 \not\subseteq \{1\}$ because 1 is not a set.

c) No, $\{2\} \notin \{1,2\}$; But $\{2\} \subseteq \{1,2\}$

d) Yes, $\{3\} \in \{1, \{2\}, \{3\}\}$

e) Yes, $1 \in \{1\}$.

f) No, $\{2\} \not\subseteq \{1, \{2\}, \{3\}\}$
Because $2 \in \{2\}$ but $2 \notin \{1, \{2\}, \{3\}\}$.

g) Yes, $\{1\} \subseteq \{1,2\}$.
$\forall x$, if $x \in \{1\}$, then $x \in \{1,2\}$.

h) No, $1 \notin \{\{1\}, 2\}$

i) Yes, $\{1\} \subseteq \{1, \{2\}\}$

$\forall x$, if $x \in \{1\}$, then $x \in \{1, \{2\}\}$.

j) Yes, $\{1\} \subseteq \{1\}$.

$\forall x$, if $x \in \{1\}$, Then $x \in \{1\}$.

Sec 1.2, #11. $A = \{w, x, y, z\}$, $B = \{a, b\}$

(b): $B \times A$ has 8 elements.

$B \times A = \{(a,w), (b,w), (a,x), (b,x),$
$(a,y), (b,y), (a,z), (b,z)\}$

[Note: The elements in $B \times A$ can be listed in any order.]

(d) $B \times B = \{(a,a), (a,b), (b,a), (b,b)\}$

$B \times B$ has 4 elements.

#12. $A = \{2, 4, 6\}$, $T = \{1, 3, 5\}$

(a) $S \times T$ has 9 elements.

$S \times T = \{(2,1)\ (2,3), (2,5), (4,1), (4,3),$
$(4,5), (6,1), (6,3), (6,5)\}$

(c) $S \times S$ has 9 elements

$S \times S = \{(2,2), (2,4), (2,6), (4,2), (4,4),$
$(4,6), (6,2), (6,4), (6,6)\}$