HW #7B , Section 6.1 Solutions

M325K SPRING 2024

Sec. 6.1

4:

$A = \{ n \in \mathbb{Z} \mid n = 5r \text{ for some integer } r \}$

$B = \{ m \in \mathbb{Z} \mid m = 20s \text{ for some integer } s \}$

Note: A is the set of all integer multiples of 5,
B is the set of all integer multiples of 20.

a) Is $A \subseteq B$? No.

Proof (NOT REQUIRED):
By def'n, $A \subseteq B \iff \forall x \in \mathbb{Z}$, if $x \in A$, then $x \in B$.

$\therefore A \not\subseteq B \iff \exists x \in \mathbb{Z}$ such that $x \in A$ and $x \notin B$.
Consider the integer 5. Now, $5 = 5 \times 1$, so $5 \in A$.
Suppose $5 \in B$ also. Then there exists an integer k such that $5 = 20k$. $\therefore \frac{5}{20} = \frac{1}{4} = k$.
Then $\frac{1}{4}$ is an integer, which contradicts the fact that $\frac{1}{4}$ is not an integer. Therefore, $5 \notin B$.
Thus, $5 \in A$ and $5 \notin B$, so $A \subseteq B$ is False.

b) Is $B \subseteq A$? Yes. Let $x \in B$ be given. Then, $x = 20s$ for some $s \in \mathbb{Z}$. $\therefore x = 5(4s) = 5t$
where $t = 4s$, which is an integer. $\therefore x \in B$.
$\therefore A \subseteq B$.

Section 6.1, #10

Let $A = \{1, 3, 5, 7, 9\}$, $B = \{3, 6, 9\}$

$C = \{2, 4, 6, 8\}$

(a) $A \cup B = \{1, 3, 5, 6, 7, 9\}$

(b) $A \cap B = \{3, 9\}$

(c) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(d) $A \cap C = \{\ \} = \emptyset$. A and C are disjoint sets.

(e) $A - B = \{1, 5, 7\}$

(f) $B - A = \{6\}$

(g) $B \cup C = \{2, 3, 4, 6, 8, 9\}$

(h) $B \cap C = \{6\}$

Sec 6.1

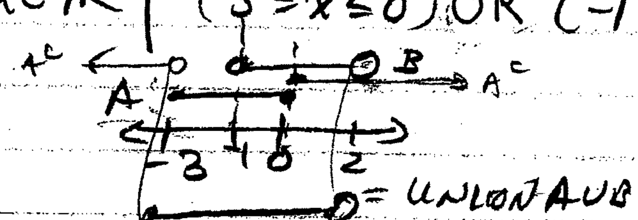
#12. $U = \mathbb{R}$.

$A = \{x \in \mathbb{R} \mid -3 \leq x \leq 0\}$

$B = \{x \in \mathbb{R} \mid -1 < x < 2\}$

$C = \{x \in \mathbb{R} \mid 6 < x \leq 8\}$

12 (a) $A \cup B = \{x \in \mathbb{R} \mid (-3 \leq x \leq 0) \text{ OR } (-1 < x < 2)\}$



(a) FINAL SOLUTION: $\boxed{A \cup B = \{x \in \mathbb{R} \mid -3 \leq x < 2\}}$

12 (b.) $A \cap B = \{x \in \mathbb{R} \mid (-3 \leq x \leq 0) \text{ AND } (-1 < x < 2)\}$

$\boxed{A \cap B = \{x \in \mathbb{R} \mid -1 < x \leq 0\}}$

12 (c) $A^c = \{x \in \mathbb{R} \mid x \notin A\}$

$= \{x \in \mathbb{R} \mid$ It is not the case that "$-3 \leq x$ and $x \leq 0$"$\}$

$= \{x \in \mathbb{R} \mid (-3 \not\leq x) \text{ OR } (x \not\leq 0)\}$

$\boxed{A^c = \{x \in \mathbb{R} \mid x < -3 \text{ OR } 0 < x\}}$

Sec 6.1 $\quad U = \mathbb{R}$.

12.(d) $\boxed{A \cup C = \{x \in \mathbb{R} \mid (-3 \le x \le 0) \text{ OR } (6 < x \le 8)\}}$

12(e)

12(f) $B^C = \{x \in \mathbb{R} \mid x \notin B\}$

$= \{x \in \mathbb{R} \mid$ It is not the case that "$-1 < x$ AND $x < 2$"$\}$

12(f) $= \{x \in \mathbb{R} \mid (-1 \not< x) \text{ OR } (x \not< 2)\}$

$\boxed{B^C = \{x \in \mathbb{R} \mid x \le -1 \text{ OR } 2 \le x\}}$

12(e) : $A \cap C = \{x \in \mathbb{R} \mid x \in A \text{ AND } x \in C\}$

$A \cap C = \{x \in \mathbb{R} \mid (-3 \le x \le 0) \text{ AND } 6 < x \le 8\}$

$A \cap C = \emptyset$ because no real number x is such that $x \le 0$ AND $6 < x$, so there is no real number such that $(-3 \le x \le 0)$ AND $(6 < x \le 8)$.

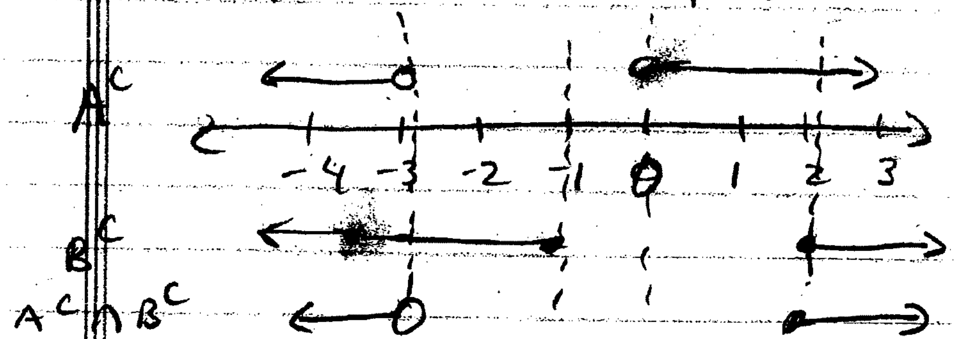
12(f) (Before the 12(e) Solution)

Sec 6.1 # 12 (g)

$A^C \cap B^C = \{x \in \mathbb{R} \mid x \in A^C \text{ AND } x \in B^C\}$

Recall: $A^c = \{x \in \mathbb{R} \mid x < -3 \text{ OR } 0 \leq x\}$

and $B^C = \{x \in \mathbb{R} \mid x \leq -1 \text{ OR } 2 \leq x\}$



(*)

$A^C \cap B^C = \{x \in \mathbb{R} \mid \text{"}x < -3 \text{ OR } 0 < x\text{" AND}$
$\text{"}x \leq -1 \text{ OR } 2 \leq x\text{"}\}$

#12(g) $\boxed{A^C \cap B^C = \{x \in \mathbb{R} \mid x < -3 \text{ OR } 2 \leq x\}}$

#12(h) Referring to the GRAPHS (*) above:

$A^C \cup B^C = \{x \in \mathbb{R} \mid \text{"}x < -3 \text{ OR } 0 \leq x\text{" OR "}x \leq -1 \text{ OR } 2 \leq x\text{"}\}$

$A^C \cup B^C = \{x \in \mathbb{R} \mid x \leq -1 \text{ OR } 0 \leq x\}$

#12(i) $(A \cap B)^c = \{ x \in \mathbb{R} \mid x \notin (A \cap B) \}$

$(A \cap B)^c = \{ x \in \mathbb{R} \mid$ It is not the case that "$-1 < x$ AND $x \leq 0$" $\}$

$(A \cap B)^c = \{ x \in \mathbb{R} \mid (-1 \nless x)$ OR $(x \nleq 0) \}$

$(A \cap B)^c = \{ x \in \mathbb{R} \mid x \leq -1$ OR $0 < x \}$

12(j) $(A \cup B)^c = \{ x \in \mathbb{R} \mid x \notin (A \cup B) \}$

$(A \cup B)^c = \{ x \in \mathbb{R} \mid$ It is not the case that "$-3 \leq x$ AND $x < 2$" $\}$

$(A \cup B)^c = \{ x \in \mathbb{R} \mid -3 \nleq x$ OR $x \nless 2 \}$

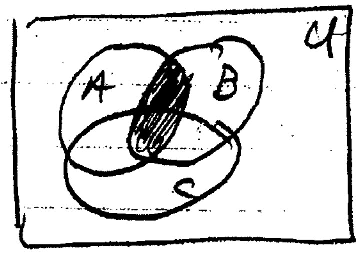
$(A \cup B)^c = \{ x \in \mathbb{R} \mid x < -3$ OR $2 \leq x \}$

#13
a. TRUE
b. FALSE
c. False
d. False $0 \notin \mathbb{Z}^- \cup \mathbb{Z}^+$.
e. True
f. True
g. TRUE
h. TRUE
i. False $\mathbb{Z} \cup \mathbb{Q} = \mathbb{Q} \neq \mathbb{Z}$

Sec 6.1 #17

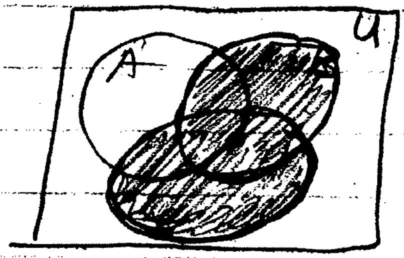
17(a)

$A \cap B$



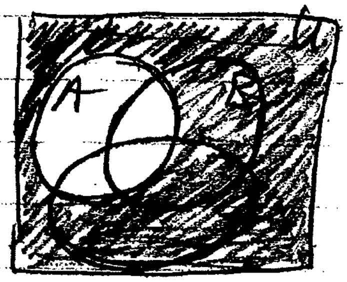
17(b)

$B \cup C$



17(c)

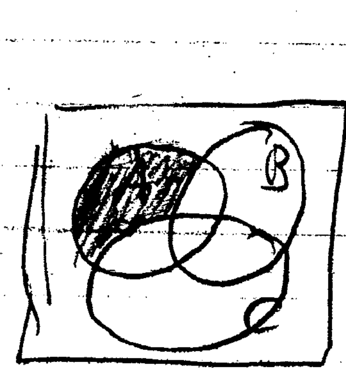
A^c



17(d) $A - (B \cup C)$

$x \in (A - (B \cup C)) \Leftrightarrow x \in A$ and $x \notin (B \cup C)$

$\Leftrightarrow x \in A$ and $x \notin B$ AND $x \notin C$



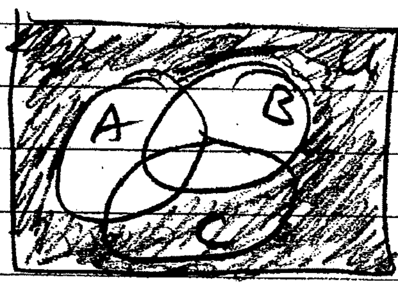
$A - (B \cup C)$

Sec 6.1

17(e) $(A \cup B)^c$

$x \in (A \cup B)^c \iff x \notin (A \cup B)$

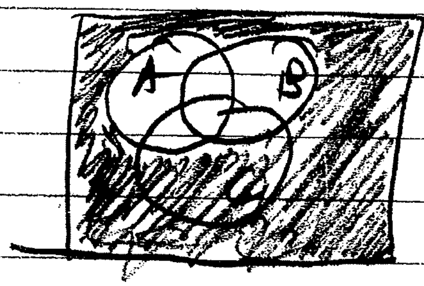
$\iff x \notin A$ AND $x \notin B$.



$(A \cup B)^c$

17(f) $x \in A^c \cap B^c \iff x \in A^c$ AND $x \in B^c$

$\iff x \notin A$ AND $x \notin B$



$A^c \cap B^c$

Note That it looks like $(A \cup B)^c = A^c \cap B^c$

This can be proven to be true for all sets A and B.

Section 6.1

#31, b) $A = \{1, 2\}$

$$P(A) = \{\{1\}, \{2\}, \{1, 2\}, \emptyset\}$$

#31, c) $A = \{1, 2\}$, $B = \{2, 3\}$

$$A \cup B = \{1, 2, 3\}$$

$$P(A \cup B) = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}, \emptyset\}$$