M325K SPRING 2024

HW#8 , Section 6.3 Solutions

Sec. 6.3

#13 STATEMENT:

FOR ALL SETS A, B, and C,

$$A \cup (B-C) = (A \cup B) - (A \cup C)$$

This Statement is false:

COUNTEREXAMPLE: Let $U = \{1,2,3,4,5,6,7,8\}$

Let $A = \{1,2,3,4\}$, $B = \{2,4,6,7\}$

$C = \{3,4,6,5\}$

$\therefore B - C = \{2,7\}$

(*) $\therefore A \cup (B-C) = \{1,2,3,4,7\}$

Now, $(A \cup B) = \{1,2,3,4,6,7\}$

$(A \cup C) = \{1,2,3,4,5,6\}$

(**) $\therefore (A \cup B) - (A \cup C) = \{7\}$,

Now $\{1,2,3,4,7\} \neq \{7\}$ Since $2 \in \{1,2,3,4,7\}$ and $2 \notin \{7\}$.

So, by substitution, $A \cup (B-C) \neq (A \cup B) - (A \cup C)$

So, The given Statement is false by proof-by-counterexample. QED.

Sec. 6.3.:

#28 (a) Set Difference Law

(b) Set Difference Law

(c) Commutative Law for "$\cap$"

(d) De Morgan's Laws

(e) Double Complement Law

(f) Distributive Laws

(g) Set Difference Law

Sec 6.3

#32 To Prove: For all sets A and B,

$$(A-B) \cup (A \cap B) = A.$$

Proof: Let A and B be any sets.

$(A-B) \cup (A \cap B) = (A \cap B^c) \cup (A \cap B)$ by The Set Difference Law

$= A \cap (B^c \cup B)$ by the Distributive Laws

$= A \cap (B \cup B^c)$ by The Commutative Laws

$= A \cap \mathcal{U}$ by The Complements Laws

$\therefore (A-B) \cup (A \cap B) = A$ by the Identity Laws.

QED, by Direct Proof.

SECTION 6.3, #34

To Prove: For all sets A, B, and C,

$$(A-B)-C = A-(B\cup C).$$

Proof: Let A, B, and C be any sets.

$$
\begin{aligned}
(A-B)-C &= (A-B)\cap C^c && \text{by the Set Difference Law,}\\
&= (A\cap B^c)\cap C^c && \text{by the Set Difference Law,}\\
&= A\cap(B^c\cap C^c) && \text{by the Associative Laws,}\\
&= A\cap((B\cup C)^c) && \text{by DeMorgan's Laws,}\\
&= A-(B\cup C) && \text{by the Set Difference Law.}
\end{aligned}
$$

$\therefore (A-B)-C = A-(B\cup C)$.

$\therefore$ For all sets A, B, and C,

$(A-B)-C = A-(B\cup C)$, by Direct Proof.

QED.

#38 To Prove: For all sets A and B, $A-(A\cap B)=A-B$.

Proof: Let A and B be sets.

$A-(A\cap B) = A\cap(A\cap B)^c$ by The Set Difference Law

$= A\cap(A^c\cup B^c)$ by DeMorgan's Law

$= (A\cap A^c)\cup(A\cap B^c)$ by the Distributive Law

$= \emptyset\cup(A\cap B^c)$ by the Complement Law for "$\cap$"

$= (A\cap B^c)\cup\emptyset$ by The Commutative Law

$= A\cap B^c$ by the identity law

$= A-B$ by the Set Difference Law.

$\therefore A-(A\cap B)=A-B$ by transitivity of "$=$".

QED, by Direct Proof.

#40 <u>To Prove</u>: For all sets A, B and C, $(A-B)-(B-C) = A-B$.

<u>Proof</u>: Let A, B and C be sets.

$(A-B)-(B-C) = (A \cap B^c) \cap (B \cap C^c)^c$ by the Set difference Law (3 times)

$= (A \cap B^c) \cap (B^c \cup (C^c)^c)$ by De Morgan's Law

$= (A \cap B^c) \cap (B^c \cup C)$ by the Double Complement Law

$= [(A \cap B^c) \cap B^c] \cup [(A \cap B^c) \cap C]$ by the Distributive Law

$= [A \cap (B^c \cap B^c)] \cup [(A \cap B^c) \cap C]$ by the Associative Law for "$\cap$"

$= (A \cap B^c) \cup [(A \cap B^c) \cap C]$ by the Idempotent Law

$= (A \cap B^c)$ by the Absorption Law

$= A - B$ by the Set Difference Law

$\therefore (A-B)-(B-C) = A-B$ by transitivity of "equals".

QED, by Direct Proof.