

HW #9, Section 8.1 Solutions

Section 8.1

#3. Define relation T from \mathbb{Z} to \mathbb{Z} as follows:

For all integers m, n, k ,

$$mTn \Leftrightarrow 3 \mid (m-n)$$

T is called the "congruence modulo 3" relation.

a) Is $10T1$?

Yes, because $10-1=9$ and $9=3 \times 3$ so

$(10-1)$ is divisible by 3.

Is $1T10$? Yes, because $1-10=-9$ and
 $-9=(-3)(3)$ and so $(1-10)$ is divisible
by 3.

Is $(2,2) \in T$? Yes, because $2-2=0$ and
 0 is divisible by 3, since $0=3 \cdot 0$.

Is $(8,1) \in T$? No, because $8-1=7$ and
 7 is not divisible by 3.

b) One possible answer is: 3, 6, 9, 12, 15

c) One possible answer is: 4, 7, 10, 13, 16

d) one possible answer is: 5, 8, 11, 14, 17.

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#4 Define Relation P on \mathbb{Z} as follows:

For all $m, n \in \mathbb{Z}$, $m P n \Leftrightarrow m$ and n have a common prime factor

a) Is $15 P 25$? Yes, since $5 | 15 = 3 \times 5$ and

$5 | 25 = 5 \times 5$, and 5 is a prime number and is a common factor of 15 and 25.

b) Is $22 P 27$? No. The only prime factors of 22 are 2 and 11 and the only prime factor of 27 is 3. So 22 and 27 have no common prime factor so $22 \not P 27$.

c) Is $0 P 5$? Yes because 5 is prime, $5 | 5$ and $5 | 0$.

d) Is $8 P 8$? Yes, because they have the common prime factor 2.

#5 $X = \{a, b, c\}$. Define R on $P(X)$ as follows:

For all $A, B \in P(X)$, $A R B \Leftrightarrow A$ and B have the same number of elements.

(a) Yes, $\{a, b\}$ and $\{b, c\}$ both have 2 elements each, so $\{a, b\} R \{b, c\}$.

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#5 (Continued)

(b) Is $\{a\} R \{a,b\}$?

No, $\{a\}$ has 1 element and $\{a,b\}$ has two elements.

c) Is $\{c\} R \{b\}$? Yes, since both $\{c\}$ and $\{b\}$ have exactly one element each.

#11. $S = \{(3,6), (4,4), (5,5)\}$

$$S^{-1} = \{(6,3), (4,4), (5,5)\}.$$

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 $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$

Define Relation V on A as follows: For all $x, y \in A$,
 $x V y \Leftrightarrow 5 \mid (x^2 - y^2)$.

Examples: $V = \{(0,5), (5,0), (1,4), (4,1), (2,6), (6,2),$
 $(3,7), (7,3), (4,4), (4,4), (7,8), (8,7),$
 $(0,0), (1,1), (2,2), (3,3), (4,4), (5,5),$
 $(6,6), (7,7), (8,8),$
 $(2,3), (3,2), (2,7), (7,2), (3,8), (8,3),$
 $(3,8), (8,3)\}$

#19 (Continued)

Here are two versions of
The Directed Graph for V :

