PROJECTED FIGURES AND TABLES ON DIRECTION FIELDS AND EULER'S METHOD

THE DIRECTION FIELD

(p. 613)

FOR

$y' = x + y$

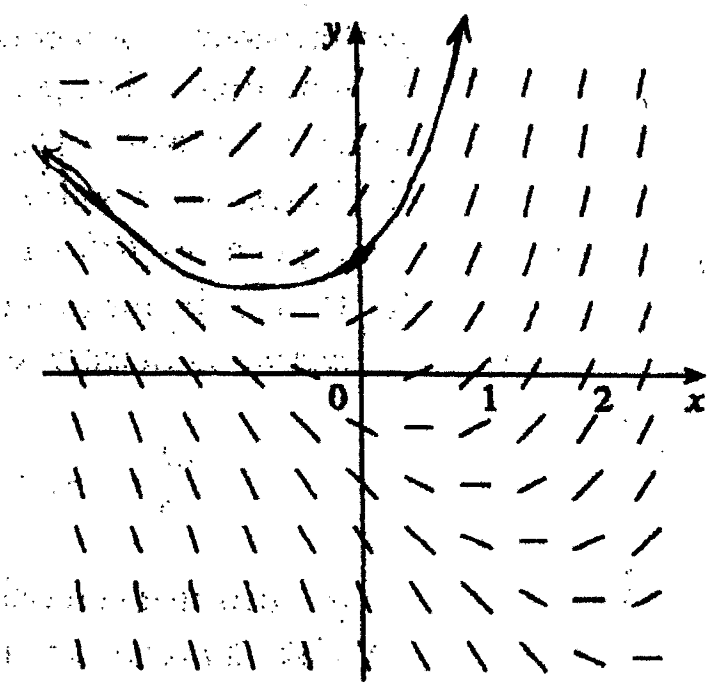


FIGURE 3
Direction field for $y' = x + y$

At any (x_0, y_0), Slope m is $m = x_0 + y_0$

Sketch the solution curve with $y(0) = 1$

THE DIRECTION FIELD FOR

$y' = x^2 + y^2 - 1$

(p. 614)

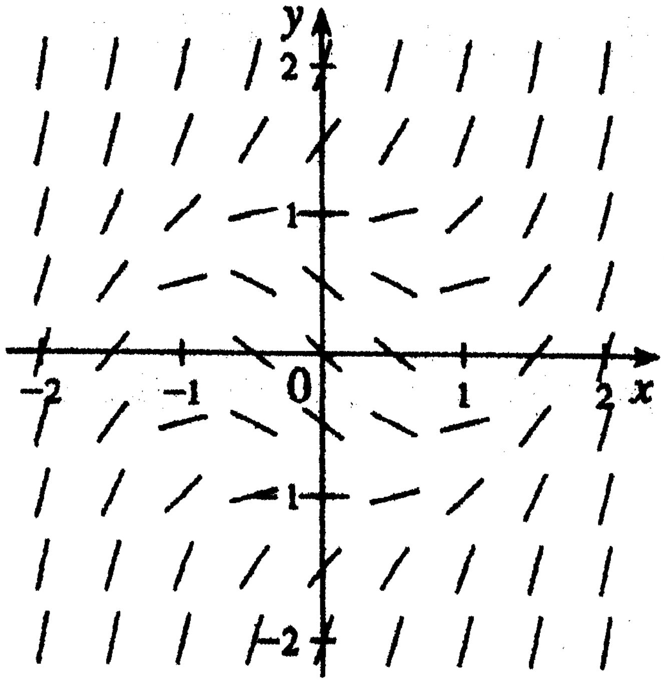


FIGURE 5

FOR

$y' = x^2 + y^2 - 1$

and

$y(0) = 0$

(p. 614)

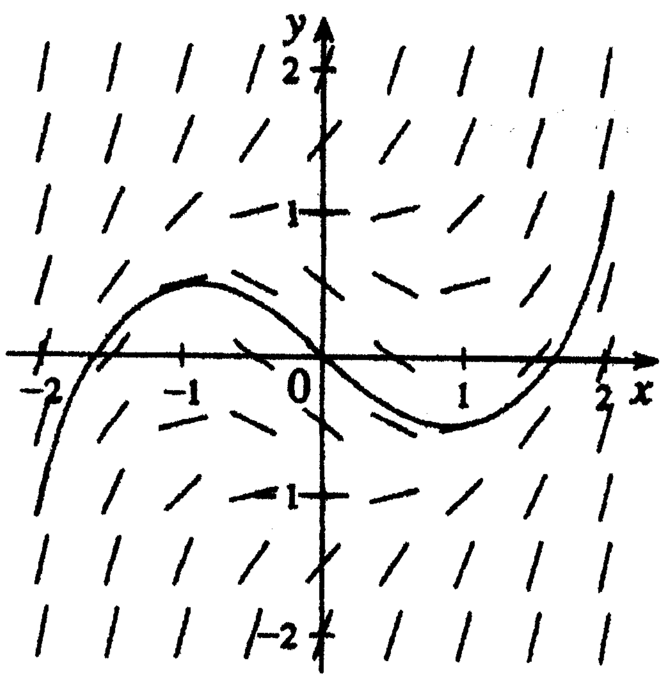


FIGURE 6

THE SLOPE FIELD
FOR
$y' = x^2 + y^2 - 1$

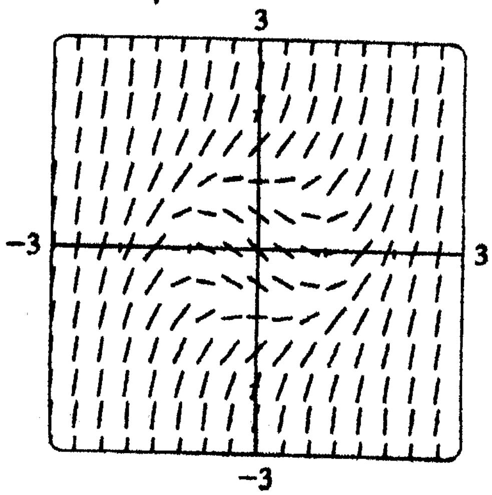


FIGURE 7
without
SOLUTION
CURVES

(p. 614)

THE SLOPE FIELD
FOR
$y' = x^2 + y^2 - 1$

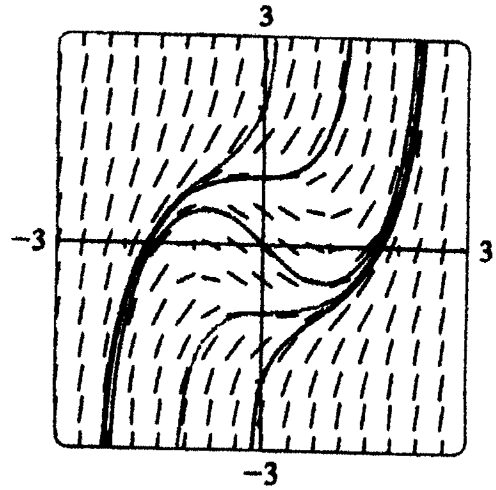
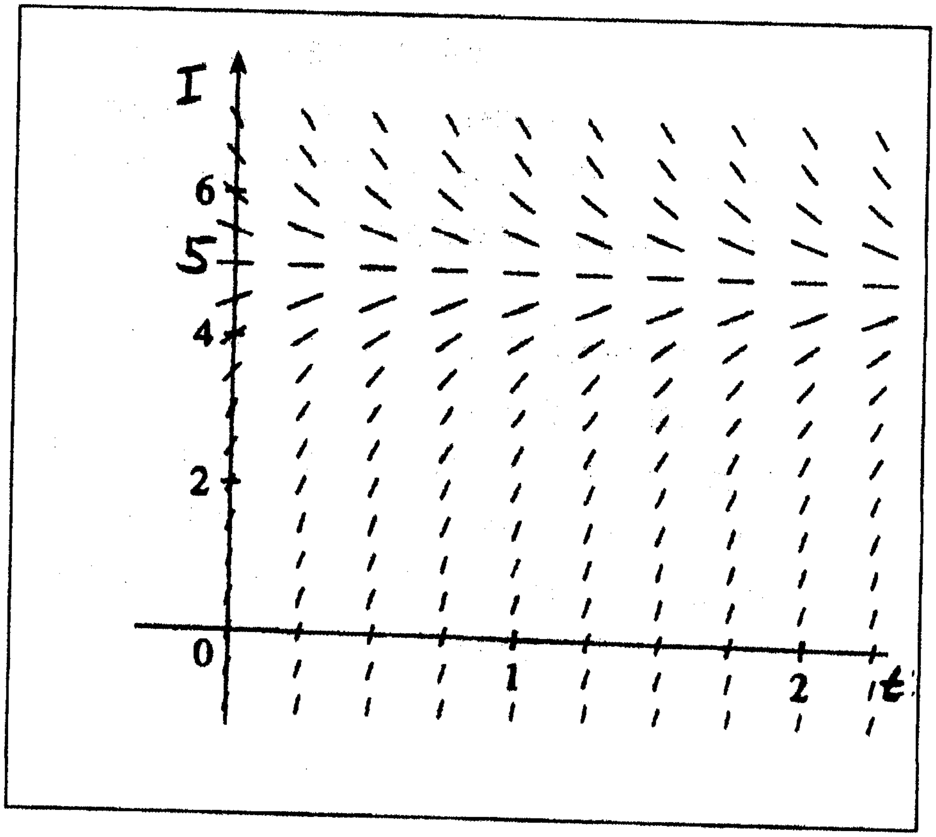


FIGURE 7

Every solution y has

$\lim_{x \to \infty} y = \infty$ and

$\lim_{x \to -\infty} y = -\infty$



FOR

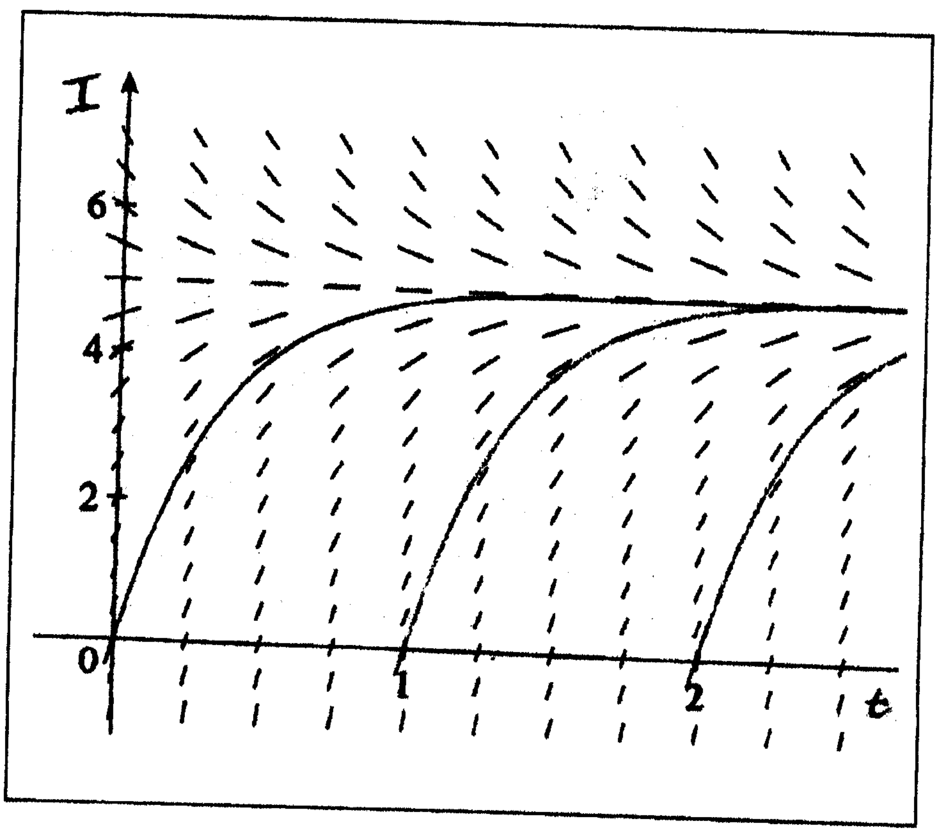
$I' = 15 - 3I$

(p. 615)

Since D.E. is $I' = f(I)$, The DE is autonomous.

Every solution I has $\lim_{t \to \infty} I(t) = 5$

$I(t) = 5$ is an equilibrium solution.



FOR

$I' = 15 - 3I$

(p. 615)

Another Autonomous D.E.

FOR $y' = 1 - y^2$

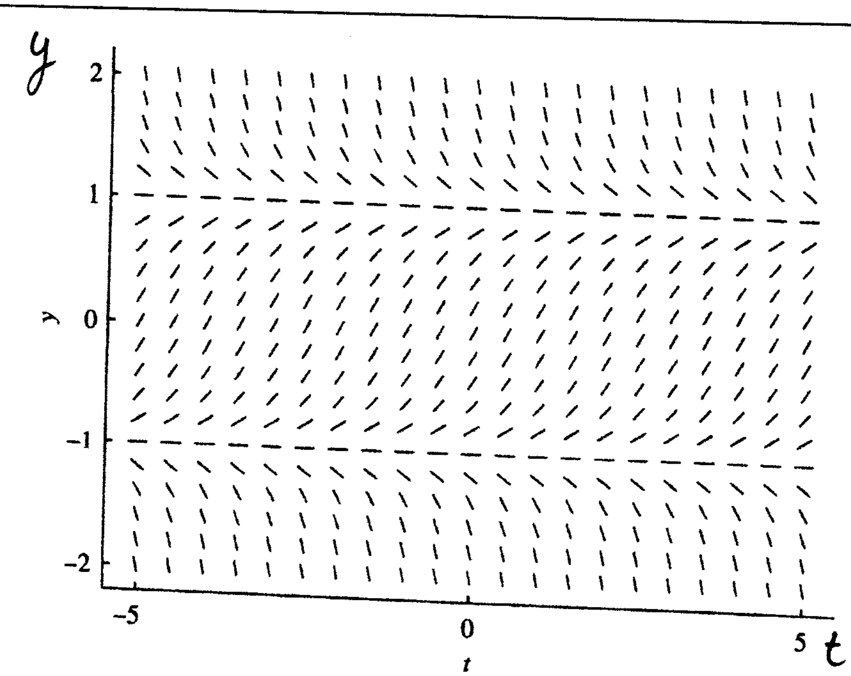
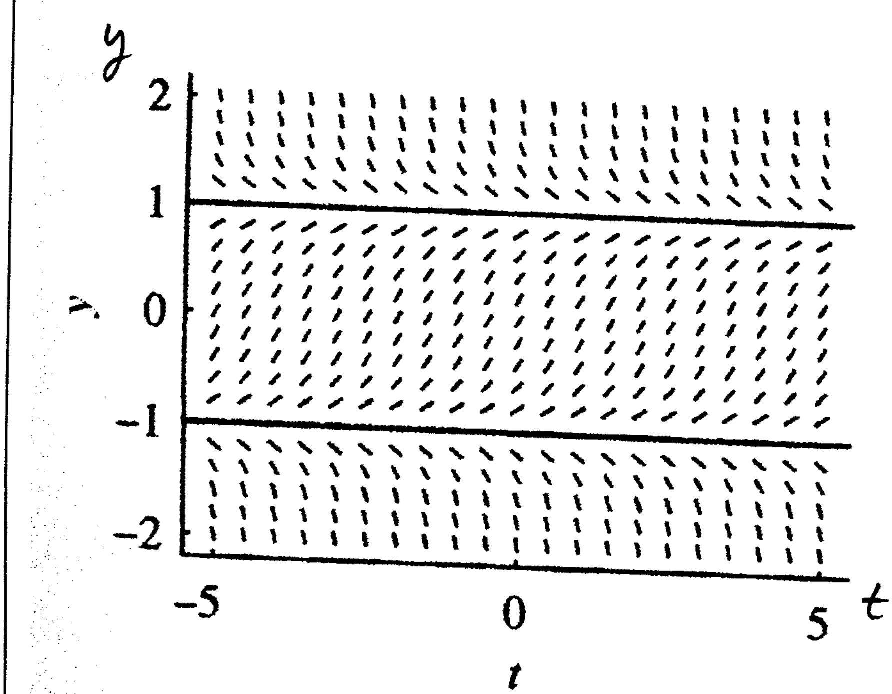


Figure 8 The direction field for the equation $y' = 1 - y^2$.

For autonomous DE $y' = f(y)$,
solve $f(y) = 0$ to find equilibrium solutions.

FOR
$y' = 1 - y^2$
$= (1+y)(1-y)$

$1 - y^2 = 0$ for
$y = -1$ and
for $y = 1$



THE DIRECTION FIELD FOR

$$y' = 1 - y^2 = (1+y)(1-y)$$

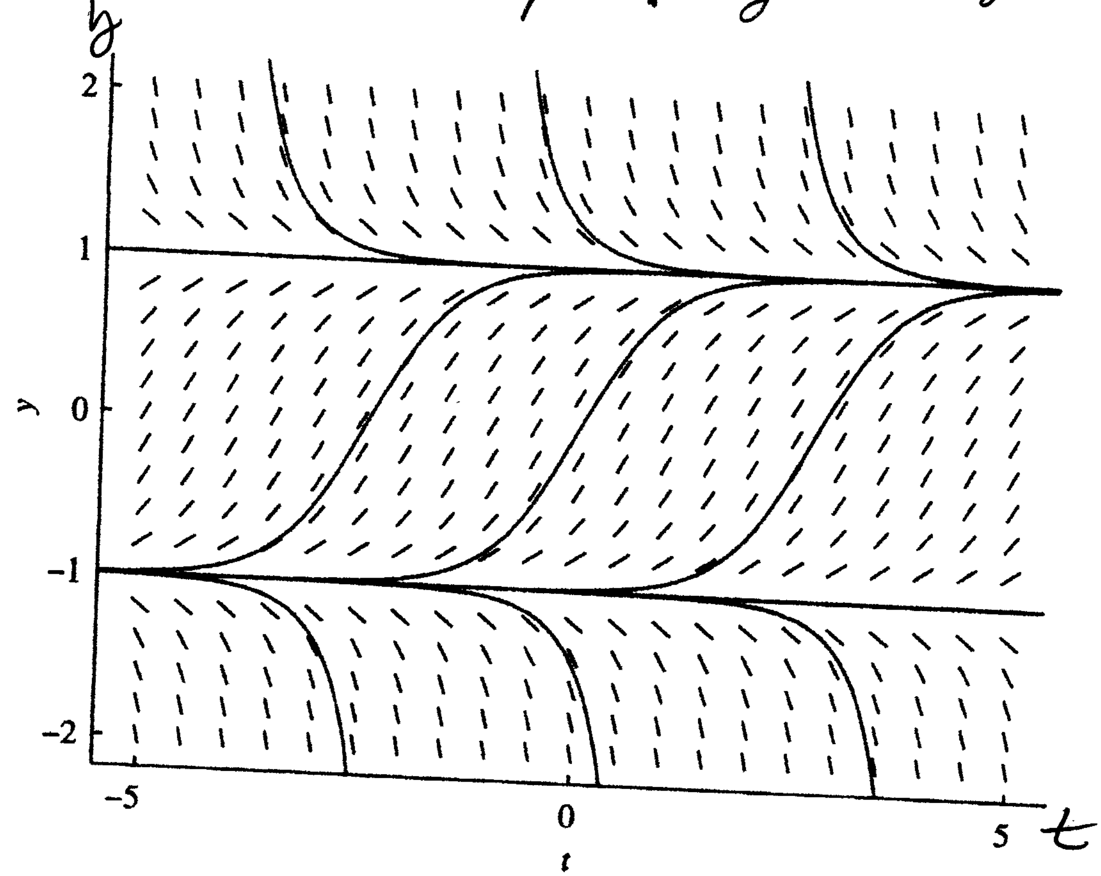


Figure 11 Typical solutions to the equation $y' = 1 - y^2$.

Solve $y' = f(y) > 0$ for $y \nearrow$

Solve $y' = f(y) < 0$ for $y \searrow$

Here: y

$y' = (+)\cdot(-) < 0$, $y \searrow$

1

$y' = (1+y)(1-y) > 0$, $y \nearrow$

$(+)\cdot(+) = (+)$

0

−1

$y' = (1+y)(1-y) < 0$, $y \searrow$

$(-)\ (+) = (-)$

EULER'S METHOD APPLIED TO THE D.E.

$y' = y$, $y(0) = 1$

with Step size $= 0.5$ to Approximate $y(2)$.

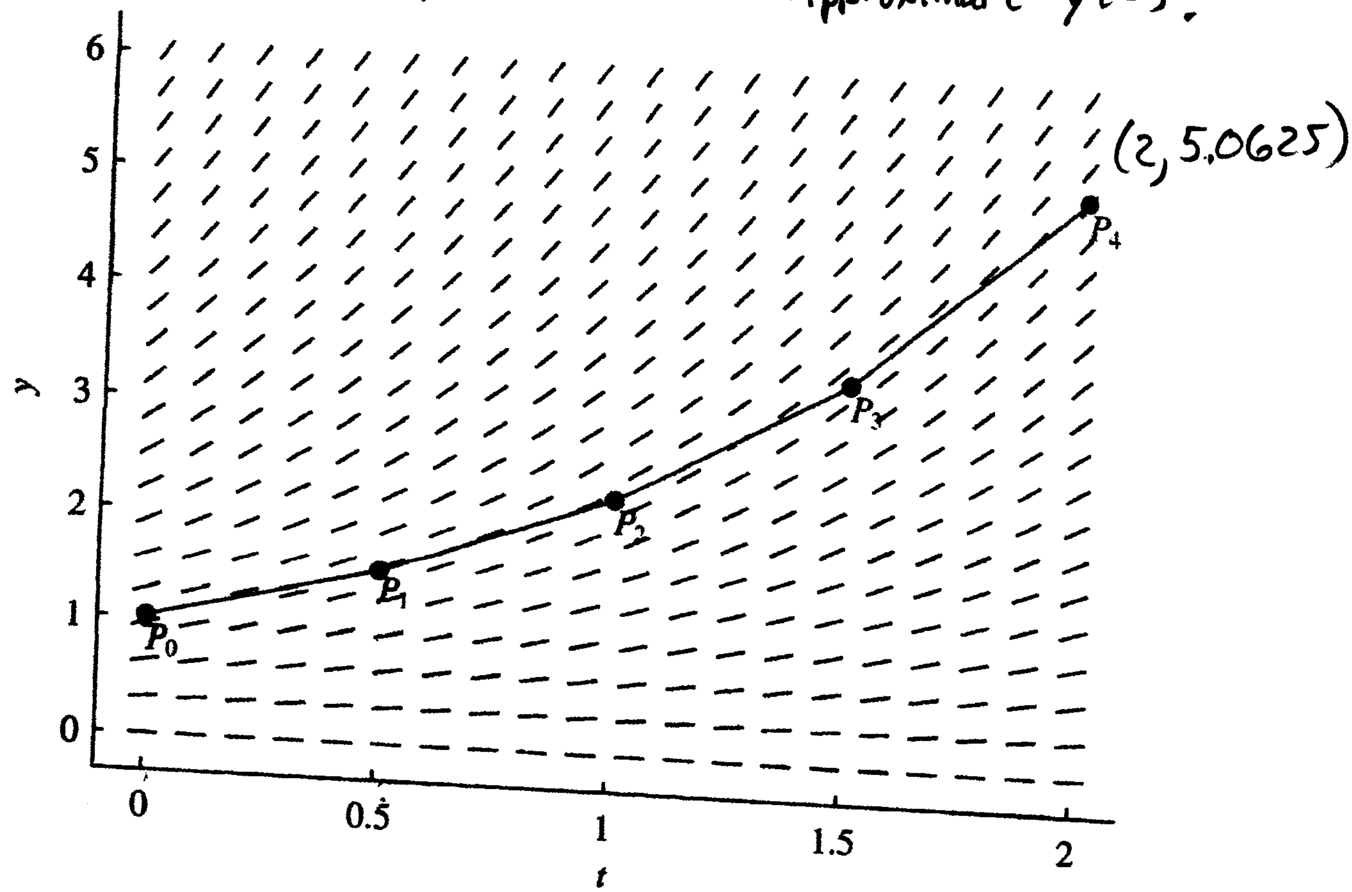


Figure 6 An approximate solution curve of $y' = y$, $y(0) = 1$.

$P_0 = (0, 1)$

$P_1 = (0.5, 1.5)$

$P_2 = (1.0, 2.25)$

$P_3 = (1.5, 3.375)$

$P_4 = (2.0, 5.0625)$

The ACTUAL SOLUTION FUNCTION is $y = e^x$

AND $y(2) = e^2 \approx 7.389056$

Actual Solution is $y = e^x$

Using Euler's Method to Approximate y(2)

Differential Equation: y' = y   Initial Values: y(0) = 1

Step Size h = 0.5   y(2) Approx. = 5.0625   Actual y(2) = 7.389056

h	n	x_{n-1}	y_{n-1}	$F(x_{n-1}, y_{n-1})$	x_n	y_n
0.5	1	0	1	1	0.5	1.5
0.5	2	0.5	1.5	1.5	1	2.25
0.5	3	1	2.25	2.25	1.5	3.375
0.5	4	1.5	3.375	3.375	2	5.0625

Using Euler's Method to Approximate y(2)

Differential Equation: y' = y   Initial Values: y(0) = 1

Step Size h = 0.1   y(2) Approx. = 6.7275   Actual y(2) = 7.389056

h	n	x_{n-1}	y_{n-1}	$F(x_{n-1}, y_{n-1})$	x_n	y_n
0.1	1	0	1	1	0.1	1.1
0.1	2	0.1	1.1	1.1	0.2	1.21
0.1	3	0.2	1.21	1.21	0.3	1.331
...	...	...	...	...	...	...
0.1	18	1.7	5.05447	5.05447	1.8	5.559917
0.1	19	1.8	5.559917	5.559917	1.9	6.115909
0.1	20	1.9	6.115909	6.115909	2	6.7275

The actual I.V.P. Solution is $y = e^x$

Using Euler's Method to Approximate y(2)

Differential Equation: $y' = y$ Initial Values: $y(0) = 1$

Step Size h = 0.01 y(2) Approx. = 7.316018 Actual y(2) = 7.389056

h	n	x_{n-1}	y_{n-1}	$F(x_{n-1}, y_{n-1})$	x_n	y_n
0.01	1	0	1	1	0.01	1.01
0.01	2	0.01	1.01	1.01	0.02	1.0201
0.01	3	0.02	1.0201	1.0201	0.03	1.030301
0.01	4	0.03	1.030301	1.030301	0.04	1.040604
...	...	...	...	...	...	...
0.01	197	1.96	7.030549	7.030549	1.97	7.100855
0.01	198	1.97	7.100855	7.100855	1.98	7.171863
0.01	199	1.98	7.171863	7.171863	1.99	7.243582
0.01	200	1.99	7.243582	7.243582	2	7.316018

Using Euler's Method to Approximate y(2)

Differential Equation: $y' = y$ Initial Values: $y(0) = 1$

Step Size h = 0.001 y(2) Approx. = 7.381676 Actual y(2) = 7.389056

h	n	x_{n-1}	y_{n-1}	$F(x_{n-1}, y_{n-1})$	x_n	y_n
0.001	1	0	1	1	0.001	1.001
0.001	2	0.001	1.001	1.001	0.002	1.002001
0.001	3	0.002	1.002001	1.002001	0.003	1.003003
0.001	4	0.003	1.003003	1.003003	0.004	1.004006
0.001	5	0.004	1.004006	1.004006	0.005	1.00501
0.001	6	0.005	1.00501	1.00501	0.006	1.006015
0.001	7	0.006	1.006015	1.006015	0.007	1.007021
...	...	...	...	...	...	...
0.001	1997	1.996	7.352223	7.352223	1.997	7.359575
0.001	1998	1.997	7.359575	7.359575	1.998	7.366934
0.001	1999	1.998	7.366934	7.366934	1.999	7.374301
0.001	2000	1.999	7.374301	7.374301	2	7.381676

The actual IVP Solution is $y = e^x$.