

Finding the Partial Fraction Decomposition (PFD) of $\frac{P(x)}{Q(x)}$

when the $\deg P(x) < \deg Q(x)$:

1) Identify the Basic Form of the PFD (factor $Q(x)$ as far as possible)

2) Write Equation: $\frac{P(x)}{Q(x)} = \text{Basic Form}$

3) Multiply by $Q(x)$ and solve for the unknowns A, B, C , etc.

Example: Let $\frac{P(x)}{Q(x)} = \frac{2x-17}{x^2-5x+4} = \frac{2x-17}{(x-1)(x-4)}$

$$\frac{P(x)}{Q(x)} = \frac{2x-17}{(x-1)(x-4)} = \frac{A}{(x-1)} + \frac{B}{(x-4)} \quad \leftarrow \text{Basic Form}$$

$$Q(x) \left[\text{-----} \right] = \left[\text{-----} \right] Q(x)$$

$$(x-1)(x-4) \left[\frac{2x-17}{(x-1)(x-4)} \right] = \left[\frac{A}{(x-1)} + \frac{B}{(x-4)} \right] (x-1)(x-4)$$

Solve for A and B in : $2x-17 = A(x-4) + B(x-1)$

Set $x = 1$: $2-17 = A(1-4) + B(1-1) \rightarrow -15 = -3A \rightarrow A = 5$

Set $x = 4$: $8-17 = A(4-4) + B(4-1) \rightarrow -9 = 3B \rightarrow B = -3$

$$\frac{2x-17}{(x-1)(x-4)} = \frac{5}{(x-1)} + \frac{-3}{(x-4)} \quad \leftarrow \text{The P. F. D.}$$