TWO ELABORATE "PARTS" EXAMPLES:

EXAMPLE 1: (multiple Applications of "PARTS")

$$\int t^2 e^{3t}\,dt = A$$

$\int u\,dv$

$$u = t^2 \qquad dv = e^{3t}\,dt$$
$$du = 2t\,dt \qquad v = \frac{1}{3}e^{3t}$$

$$A = uv - \int v\,du = \frac{1}{3}t^2e^{3t} - \int\left(\frac{1}{3}e^{3t}\right)(2t\,dt)$$

$$= \frac{1}{3}t^2e^{3t} - \frac{2}{3}\int t e^{3t}\,dt = B$$ (use "PARTS" AGAIN!)

$$u = t \qquad dv = e^{3t}$$
$$du = 1\,dt \qquad v = \frac{1}{3}e^{3t}$$

$$B = \frac{1}{3}t^2e^{3t} - \frac{2}{3}\left[\frac{1}{3}te^{3t} - \int \frac{1}{3}e^{3t}\,dt\right]$$

$$= \frac{1}{3}t^2e^{3t} - \frac{2}{9}te^{3t} + \frac{2}{9}\int e^{3t}\,dt$$

$$= \frac{1}{3}t^2e^{3t} - \frac{2}{9}te^{3t} + \frac{2}{27}e^{3t} + C$$

$$= \left[\frac{1}{3}t^2 - \frac{2}{9}t + \frac{2}{27}\right]e^{3t} + C$$

THUS:
$$\int t^2e^{3t}\,dt = \left[\frac{1}{3}t^2 - \frac{2}{9}t + \frac{2}{27}\right]e^{3t} + C$$

EXAMPLE 2 : (Applying "PARTS" TWICE TO RETURN TO AN INTEGRAL RELATED TO THE ORIGINAL SO THAT YOU CAN SOLVE FOR IT!)

$$\int e^x \cos x \, dx = A$$

$$\left[\begin{array}{ll} u = e^x & dv = \cos x \, dx \\ du = e^x dx & v = \sin x \end{array}\right]$$

$$A = e^x \sin x - \int e^x \sin x \, dx = B$$

$$\left[\begin{array}{ll} u = e^x & dv = \sin x \, dx \\ du = e^x dx & v = -\cos x \end{array}\right]$$

$$B = e^x \sin x - \left[-e^x \cos x - \int -e^x \cos x \, dx \right]$$

$$= e^x \sin x + e^x \cos x - \int e^x \cos x \, dx .$$

THUS: $\int e^x \cos x \, dx = e^x \sin x + e^x \cos x - \int e^x \cos x \, dx$

ADD!

$$2 \int e^x \cos x \, dx = e^x \sin x + e^x \cos x + C$$

Divide!

$$\int e^x \cos x \, dx = \frac{1}{2} e^x \sin x + \frac{1}{2} e^x \cos x + C$$

$$\int e^x \cos x \, dx = \frac{1}{2} e^x (\sin x + \cos x) + C$$