Finding the Maclaurin Series for $f(x) = \cos x$

The series $\sum_{n=0}^{\infty} c_n x^n$ has $c_n = \dfrac{f^{(n)}(0)}{n!}$.

	n	$f^{(n)}(x)$	$f^{(n)}(0)$	c_n	$c_n x^n$	
✓	0	$\cos x$	1	$1/0! = 1$	$1 = x^0/0!$	$m=0$
	1	$-\sin x$	0	$0/1! = 0$	$\cancel{0x}$	
✓	2	$-\cos x$	-1	$\frac{-1}{2!} = -\frac{1}{2}$	$-x^2/2$	$m=1$
	3	$\sin x$	0	$0/3! = 0$	$\cancel{0x^3}$	
✓	4	$\cos x$	1	$1/4!$	$+\ x^4/4!$	$m=2$
	5	$-\sin x$	0	$0/5! = 0$	$\cancel{0x^5}$	
✓	6	$-\cos x$	-1	$-1/6!$	$-\ x^6/6!$	$m=3$
	7	$\sin x$	0	$0/7! = 0$	$\cancel{0x^7}$	
✓	8	$\cos x$	1	$1/8!$	$+\ \frac{x^8}{8!}$	$m=4$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	

$$\cos x = \underbrace{\frac{x^0}{0!}}_{m=0} + \underbrace{(-1)\frac{x^2}{2!}}_{m=1} + \underbrace{\frac{x^4}{4!}}_{m=2} + \underbrace{(-1)\frac{x^6}{6!}}_{m=3} + \underbrace{\frac{x^8}{8!}}_{m=4} + \cdots$$

$$\cos x = \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{(2m)!} \xrightarrow{\text{Write using } n} \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

So, $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ = The Maclaurin series for $\cos x$

We Determine The Radius of Convergence R using The ratio test:

$$\cos x = \sum_{n=0}^{\infty} a_n \quad \text{where} \quad a_n = \frac{(-1)^n x^{2n}}{(2n)!}.$$

$$\frac{|a_{n+1}|}{|a_n|} = \frac{|x|^{2(n+1)}}{(2(n+1))!} \cdot \frac{(2n)!}{|x|^{2n}}$$

$$\frac{|a_{n+1}|}{|a_n|} = \frac{|x|^{2n+2}}{(2n+2)!} \cdot \frac{(2n)!}{|x|^{2n}}$$

$$\frac{|a_{n+1}|}{|a_n|} = \frac{|x|^2}{(2n+2)(2n+1)} < \frac{|x|^2}{(2n)(2n)} = \frac{|x|^2}{4n^2} \to 0$$

as $n \to \infty$.

$$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{|x|^2}{(2n+2)(2n+1)} = 0 = L < 1$$

Thus, $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ converges absolutely for all x in $\mathbb{R}$. The I.O.C is $(-\infty, \infty)$ and The Radius of Convergence is $\underline{R = \infty}$