# FINDING the Maclaurin Series for $f(x) = e^x$

Recall, here $a=0$ and $c_n = \dfrac{f^{(n)}(0)}{n!}$

| $n$ | $f^{(n)}(x)$ | $f^{(n)}(0)$ | $c_n$ | $c_n x^n$ |
|---|---|---|---|---|
| 0 | $e^x$ | $e^0 = 1$ | $e^0/0! = 1$ | $1 = x^0/0!$ |
| 1 | $e^x$ | $e^0 = 1$ | $e^0/1! = 1$ | $x = x^1/1!$ |
| 2 | $e^x$ | $e^0 = 1$ | $e^0/2! = 1/2!$ | $\frac{1}{2}x^2 = x^2/2!$ |
| 3 | $e^x$ | $e^0 = 1$ | $e^0/3! = 1/3!$ | $\frac{1}{6}x^3 = x^3/3!$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |

The Maclaurin series for $f(x) = e^x$ is:

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = \sum_{n=0}^{\infty} a_n \quad \text{where } a_n = \frac{x^n}{n!}$$

To find the RADIUS OF CONVERGENCE $R$, we use the ratio test:

$$\frac{|a_{n+1}|}{|a_n|} = \frac{|x|^{n+1}}{(n+1)!} \cdot \frac{n!}{|x|^n} = \frac{|x|}{n+1}.$$

For any $x$ in $\mathbb{R}$,

$$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{|x|}{n+1} = 0 < 1,$$ so the

Maclaurin Series for $f(x) = e^x$ converges absolutely for all $x$ in $\mathbb{R}$.

The RADIUS OF Convergence is $R = \infty$.