THE TAYLOR POLYNOMIALS $T_n(x)$ AND TAYLOR'S INEQUALITY

THE TAYLOR POLYNOMIALS $T_n(x)$

Given function $f(x)$ and

its Taylor Series at a: $\displaystyle\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$,

Then,

The "n^{th} Taylor Polynomial of f at a" is

$$T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k .$$

TAYLOR'S INEQUALITY

Regarding the $(n+1)^{st}$ derivative of f,

and regarding positive constants M and d,

If $|f^{(n+1)}(x)| \leq M$ for all #'s x with

$$|x-a| \leq d,$$

Then the remainder $R_n(x)$ of the Taylor Series centered at a satisfies the inequality:

$$|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1} \text{ for all } |x-a| \leq d .$$